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Editorial

Nehru Arts & Science College Kanhangad, affiliated to Kannur University, has entered into the 50th year of its effective functioning. As an educational institution it imparts University level education to students in Science, Social Science, Management and Language streams in the relatively backward district of Kasaragod in Kerala. The institution has made important contribution in the fields of teaching, learning, research and extension activities. *Academia* is a multidisciplinary research journal and the present volume is a rich collection of original research papers of the teachers pertaining to this institution. The journal fosters academic writing and engages in dissemination of knowledge. I profusely thank all the paper contributors for enriching the volume.

A.Ashokan

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Theories of Regional Economic Development: An Analytical Survey

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Abstract

The aim of the paper is to provide a brief analytical survey of some of the well received theories of development. These theories encourage us to assess regional development from a national perspective. The paper unfolds a case for framing appropriate regional development policies by identifying the key drivers in the development process.

During the 1950s and 1960s, some theories were developed to investigate the determinants of development and viewed it as largely based on the growth of a region's output or per capita income. These theories further accept that the factors which may trigger a growth process include increased demand for locally produced goods, greater local production capacity, a more abundant endowment of local resources and production factors, larger investments in infrastructures, technology, efficiency of production processes, etc. The significant determinants of regional development include: human capital, innovation and diffusion of innovation produced elsewhere, entrepreneurial ability, local resources, the decision making capacity of local economic and social actors, infrastructural facilities, implementation of regional development agenda, role of institutions, the strength of both forward and backward linkages, labour productivity, strength and size of market, geography, structuralism, etc. From this perspective, economic development is accomplished by designing spatial level intervention through appropriate regional policy initiatives including decentralization through multi-level governance. The basic theory and philosophy of economics and regional economics are almost identical. Regional economics is a focused enquiry which examines spatial economic problems. For instance, resource mobilization as well as its efficient utilization is the major issue in any branch of economic analysis. Following this, regional economic development aims at exploiting regional scale economies through appropriate development strategies. Further, innovation and growth are rooted in specific places or regions, rather than countries. The present paper is an analytical survey of the theories of (regional) development which would be useful to students, policy makers and development practitioners.

The regional specificities are crucial because economic activity originates, grows and ultimately sustains in space. Differences in geographical distribution of resources and economic activities account for differences in costs of production, factor payments, wealth and well-being. These factors either promote or restrict the development process. Economic transformation, in general, is influenced by many economic and non-economic factors both inside and outside the economic system. That is, growth is influenced by a number of factors grouped into either endogenous or exogenous factors. In endogenous growth theory the rate of growth is determined by the equilibrium solution of the growth model itself and technical progress is explicitly

modelled. It holds that investment in human capital, innovation, knowledge, entrepreneurial ability, mobilization of local resources for production (including labour and capital), the decision-making capacity of local economic and social actors able to control the development process are significant contributors to economic growth.On the other hand, exogenous factors comprise of the fortuitous local presence of a dominant firm or a multinational enterprise, the diffusion in the area of an innovation produced elsewhere, the implementation of new infrastructures decided by external authorities.

The theories or strategies or models provide a broad perspective about the issues involved in development or underdevelopment. For instance, William Arthur Lewis proposed that there is disguised unemployment or underemployment in rural areas of poorer countries, where the marginal productivity of labour is, in general, very low. Transfer of labour from rural to urban sectors will lead to surplus labour as a saving potential. That is, migrants to the city will be placed in more productive jobs and will pull their families out of poverty. Hans Singer and Raul Prebisch further argued that there is a long-run tendency for the terms of trade to move against primary goods. Urban planners, meanwhile, argue that goods and services would most efficiently be diffused from major cities to smaller cities and rural areas. In the 1960s Michael Lipton began to develop a more 'positive' (or scientific) account of urban bias in the process of world development. The Urban Bias Thesis (UBT) proposes that urban classes in poorer countries use their social power to bias (distort) a range of public policies against members of the rural classes. The Urban bias thesis by Lipton identified the following key elements: i) Rural areas of developing countries suffer from too little spending on education and health care (relative to population size); ii) These inequalities, combined with excessively urban form of teaching and curriculum development pull bright young people to the cites; iii) People in rural areas are forced to pay a higher share of national taxes than what is warranted; iv) Urban bias is further and most damagingly evident in a series of government imposed price twists, which costs inputs into rural areas to be overpriced as compared with a market norms and cause output from rural areas to be correspondingly underpriced. International urban biases-a range of forces in the international economic system also effectively discriminate the agricultural sector in developing countries. There are well known development theories in the Classical, Marxian, Neo-classical, Keynesian, Post Keynesian, Institutional and other traditions. This paper is an attempt to trace some of the important theories of (regional) development.

1. Cumulative Causation Model

It is an approach to the analysis of regional economic growth developed mainly by Gunnar Myrdal. He challenges the neoclassical proposition that unfettered trade between regions would ultimately lead to a reduction in inequalities across regions. The theory states that market forces tend to increase economic inequalities between regions of the same economy. He argues that increasing returns to scale results in the clustering of economic activity within regions that are the first to industrialize. The basic argument is that the process of growth tends to feed on itself through a process that he describes as cumulative causation. To him, in Less Developed Countries a circular and cumulative causation operates downwards and being unregulated causes increasing inequalities. He constructs his theory of economic development or underdevelopment around the idea of regional inequalities on the national and international levels. He uses the concepts of backwash and spread effects to explain his thesis. Backwash effects are all relevant adverse changes of economic activity in a locality ... caused outside that locality. This takes place through migration, capital movements and trade as well as the total cumulated effects resulting from the process of circular causation between both 'economic' and 'non-economic' factors. The spread effects refer to certain centrifugal spread effects of expansionary momentum from the centres of economic expansion to other regions. The main cause of regional inequalities, to Myrdal, has been the strong backwash effects and the weak spread effects.

Under the neo-liberal regime, the major development challenge is the yawning gap between the affluent and the poor and economic inequalities that gets accentuated (Stiglitz, 2012). According to circular and cumulative causation perspective, development and under-development are self-reinforcing. However, comparative development depends on the

relative weight of unequalising centripetal forces of attraction and equalising centrifugal forces of diffusion. He further argues that regional inequalities are associated with the capitalist system guided by profit motive. The profit motive results in the development of those regions where the expectations of profits are high while other regions remain underdeveloped. This is attributed to the free play of market forces which tends to increase rather decrease regional inequalities. He states that "if things were left to market forces uninfluenced by any policy interferences almost all major economic activities would cluster in certain localities and regions, leaving the rest of the country more or less in a backwater" (Myrdal,1957). Migration, capital movements and trade strengthen the localities and regions where economic activity favours and expands the developing or developed regions and depress economic activity in the backward region. Similarly, manufacturing and agricultural production will be differentially affected in developed and underdeveloped regions. Thus, the major cause of the backwardness of UDCs has been the weaker spread effects and stronger backwash effects whereby in the cumulative process "poverty becomes its own cause". In the highly integrated global economic system, trade and capital flow further widens inequality and unequal exchange. Keynesian demand management is an effective option to level off inequality in consumption and quality life including health education. That is state may act as an instrument of regional development in poor countries. The free play of market forces and the laissez faire policy have been accentuating regional inequalities through weaker spread effects. Regional inequalities in poor countries get further accentuated through built-in feudal and other inegalitarian institutions and power structures which aid the rich in exploiting the poor. The governments of LDCs should, therefore, adopt egalitarian development policies to weaken the backwash effects and strengthen the spread effects in order to bridge regional inequalities and to strengthen the foundations for continuous economic progress. This is a process through which a national state becomes a welfare state.

International trade may have strong backwash effects on the LDCs. From a political economy angle trade leads to unequal exchange. "Trade operates (as a rule) with a fundamental bias in favour of the richer and progressive regions (and continues) and in disfavour of the LDCs". Trade between industrialized countries and LDCs strengthens the former and impoverishes the latter. This is attributed to the different commodity producing bases of the two countries. Further capital movements have failed to counteract international inequalities and widened the economic gap between the rich and poor countries. International migration between the LDCs and developed countries is not a sustainable solution to the problem of international inequalities.

2. Hirschman's Strategy of Unbalanced Growth

A.O. Hirschman argues that a deliberate unbalancing of the economy through a pre-designed strategy is the best method of development. To quote him, "development is a chain of disequilibria that must be kept alive rather than eliminate the disequilibrium of which profits and losses are the symptoms in a competitive economy. If the economy is to be kept moving ahead, the task of development policy is to maintain tensions, disproportions, and disequilibria. Therefore, the sequence that leads away from equilibrium is precisely an ideal pattern of development from our point of view for each move in the sequence is induced by a previous disequilibrium and in turn creates a new disequilibrium that requires a further move. The path of unbalanced growth is described by the three phrases "induced investment, complementarity, and external economies". In the typical growth context, induced investment strengthens scale economies, both internal and external.

Social overhead capital (SOC) and directly productive activities (DPA) are the two major determinants of development. SOC is usually defined as "comprising those basic services without which primary, secondary, and tertiary productive activities cannot function". It includes all public services such as law and order, education, public health, transport, communication, power, water supply, irrigation, and drainage systems, etc. Hirschman has discussed the strategy of unbalanced growth under the following two headings: a) Development via shortage of SOC; and b) Development via excess capacity of SOC. In the former case, the country in question invests in DPA first. Because of the

expansion of DPA that results due to such an investment, the pressure on SOC starts building up. This induces investment in SOC. In the case of development via excess capacity of SOC, expansion of SOC reduces the cost of such services like transportation, power, etc and this discourages investment in DPA.

He offers an alternative to the prevailing balanced growth paradigm which essentially states that economic development requires the simultaneous development of a large number of new industries to support the linkages required for steady growth. He examines the theory of economic development based on the concept of backward and forward linkages among firms. Backward linkage exhibits the relationship between an industry or firm and the suppliers of its inputs. A change in the output of the industry will be transmitted backwards to the suppliers of its inputs by a change in demand for inputs. In other words, for a given firm, backward linkages refer to the output to the inputs of other firms used in the production process. On the other hand, forward linkage unfolds the relationship between an industry or firm and other industries or firms which use its output as an input. A change in output or price will be transmitted forwards to users of its product. In other words, forward linkages refer to the output of the firm that is then used by other local producers as an intermediate good for the production of other products. Hirschman's concepts of directly productive activities and overhead capital can be extended to develop a framework for determining the regional improvements in infrastructure. Further, the impact of new investments on future development varies according to the existing level of development in a region. In "congested" regions with high levels of productive activity compared to the level of infrastructure, the region may require investments targeted at specific congestion problems without general expansions of the entire infrastructure network. In intermediate regions with high resource potential but deficiencies of core infrastructure, policy may demand across the board expansions of transportation, communications and power infrastructure. Finally, "lagging" regions with a shortage of physical and human capital would most likely benefit from investments in social services such as health care and education in preparation for the eventual expansion of productive capacity.

3. Export base theory

One of the most explicit theories of economic development is the demand-oriented export base theory. This theory argues that a country's or community's economy may be bifurcated into two sectors: an export or basic sector and a non-export or non-basic sector. The basic sector that trades outside its boundaries brings money into the local economy, which stimulates future economic development. The non-basic sector, on the other hand, supplies local consumption of goods and services whose activity depends upon basic sector export sales. External demand for a region's exportable goods and services injects income into the local economy, which in turn encourages local demand for non-exportable goods and services. One of the uses of export-base theory is the identification of economic sectors that export and the quantity of their export sales.' By identifying the export or basic sectors, regional development practitioners can identify factors that influence export sales. If some factors are endogenous to the regional economy, regional development authorities may be able to formulate strategies to strengthen, protect, or expand sectoral export sales. Sectors that do not export and that may be importers of a given good and service also can be identified. By identifying importing sectors, regional economic development practitioners can formulate import substitution strategies that potentially could reverse flow of money from the regional economy. Probably the most expansive use of export-base theory is the development of export-base multipliers for impact analysis. The estimation of sectoral basic and non-basic employment or income is essential for the estimation of export-base multipliers. One of the most contentious issues facing development of an export-base model is the identification of a region's basic or export activity. The economic base model is predicated upon the bifurcation of regional economic activity into at least two distinct sectors, export or basic sector and non-export or non-basic sector. State and country data for development of export-base activity usually cannot be obtained, except at a high cost. Because of the high cost, potential selection bias and accuracy errors of respondents, many regional scientists and development practitioners have adopted non-survey techniques for identifying basic or export activity. Numerous

studies have appeared in the literature devoted to the formulation and refinement of competing non-survey techniques. Export base theory has been one of the most used theories of economic development to describe a local economy and target sectoral economic development strategies. A criticism of export-base theory is the identification of basic and non-basic activity for a sector or a region. Several indirect procedures such as the assignment, location quotient, and minimum requirements procedures have been employed to estimate basic sector activities. All of these procedures have shortcomings in the estimation of basic sector activity.

4. Central Place Theory

The Central Place theory was first developed by the German geographer Walter Christaller in 1933. He began to recognize the economic relationships between cities and their hinterlands. He tested the theory in Southern Germany and came to the conclusion that people gather together in cities to share goods and ideas and that they exist for purely economic reasons. The explanation of the existence of urban systems made up of cities of different sizes is due to the school of the "central place theory". The founders of this school of thought were the geographer Walter Christaller and the economist August Lsch. These were the first to formulate models to explain the urban hierarchy, and in particular: the size and frequency of urban centres at every level in the hierarchy, and therefore the market area of each of them; the distance between a particular city and those at the levels immediately below or above it, and therefore the geographical distribution of all the urban centres. These models put forward a more complex and general theory of location and the structure of the underlying economic relations able to account for the existence of diverse territorial agglomerations within a framework of spatial equilibrium. The central place exists primarily to provide goods and services to its surrounding population. The city is in essence, a distribution centre. Two assumptions were made about human behavior: 1) Humans will always purchase goods from the closest place that offers the good; and 2) Whenever demand for a certain good is high, it will be offered in close proximity to the population. When demand drops, so does the availability of the good. In addition, threshold is an important concept in Christaller's study. This is the minimum number of people needed for a central place business or activity to remain active and prosperous.

Central Place Size and Spacing

Within the central place system, there are five sizes of communities. A hamlet is the smallest and is a rural community which is too small to be considered a village. The rank order of central places is:
Hamlet: fewest goods and services available
Village: includes the region of the hamlet and some additional goods and services
Town: includes the region of the village and hamlet and provides some additional goods and services
City: includes the region of the village, hamlet and town and provides additional goods and services
Regional Capital

Central-place theory,in geography, an element oflocation theoryconcerning the size and distribution of central places (settlements) within a system. Central-place theory attempts to illustrate how settlements locate in relation to one another, the amount ofmarketarea a central place can control, and why some central places function as hamlets, villages, towns, or cities. The primary purpose of a settlement or market town, according to central-place theory, is the provision of goods and services for the surrounding market area. Such towns are centrally located and may be called central places. Settlements that provide more goods and services than do other places are calledhigher-order central places.Lower-order central places have small market areas and provide goods and services that are purchased more frequently than higher-order goods and services. Higher-order places are more widely distributed and fewer in number than lower-order places. Christaller's theory assumes that central places are distributed over a uniform plane of constant population density and purchasing power. Movement across the plane is uniformly easy in any direction, transportation costs vary linearly, and consumers act rationally to minimize transportation costs by visiting the nearest location offering the desired good or service. The determining factor in the location of any central place is thethreshold, which comprises the smallest market area necessary for the goods and services to be economically viable. Once a threshold has been

established, the central place will seek to expand its market area until therangei.e.,the maximum distance consumers will travel to purchase goods and servicesis reached. Since the threshold and range define the market area of a central place, market areas for a group of central places offering the same order of goods and services will each extend an equal distance in all directions in circular fashion. The German economistAugust Lsch expanded on Christaller's theory. Unlike Christaller, whose system of central places began with the highest-order, Lsch began with a system of lowest-order (self-sufficient) farms, which were regularly distributed in a triangular-hexagonal pattern. He also illustrated how some central places develop into richer areas than others.

5. Sector Theory (Kuznets)

The sector theory has its origin in the empirical observations made by Colin Clark, Simon Kuznets and others. It is based on the contribution of different sectors of economy at different levels of development. The sector theory places attention on structural changes taking place within an economy in contrast to the export base theory, which emphasizes the role of external relationships. According to sector theory, the process of economic development is accompanied by a shift in the employment pattern first from primary to secondary sector and later on to the tertiary sector. The explanation is based upon different income elasticity for the products of these sectors and the relative differences in the average earnings per worker in different sectors. The theory is empirically verifiable in terms of cross section and historical trends in different countries or major regions within them. The sector theory with its emphasis on structural changes, differences in elasticity of demand and productivity differences among sectors throws light on some important elements in the growth of an economy. It provides a useful frame of reference for aggregating data for comparative studies. However, the main weakness of the sector theory is its neglect of the role of external factors in regional development. Reliance on aggregation of data at a very broad level also has its limitations.

6. Stages of Growth Theory (Rostow)

This is one of the important historical theories of economic growth, suggested by Prof.W.W.Rostow. He divides the growth process into five stages:

1. **The Traditional Society:** It is a less developed society in terms of both science and technology. However, the concept of traditional society is not wholly static. During this period some cultivable land was still available for agriculture. Irrigation methods were known and both farmers and the state expanded these facilities. According to Rostow, "the ceiling resulted from the fact that the potentialities which flow from modern science and technology were either not available or not regularly and systematically applied".

2. **The Pre-conditions for take-off:** These relate to the application of modern science to agriculture. Europe at the end of the 17th century is stated as an example.

3. **The Take- off:** It is a decisive stage in the evolution of any society. In this stage growth becomes a normal condition of society. The take-off process generally begins on account of some sharp stimulus which may come from a political revolution, a technological innovation or even a favourable international environment. Rostow identifies the following three conditions: "a) a rise in the rate of productive investment from, say 5 percent or less to over 10 percent of national income (or NNP); b) the development of one or more substantial manufacturing sectors, with a high rate of growth; c) the existence or quick emergence of a political, social and institutional framework which exploits the impulses to expansion in the modern sector and the potential external economy effects of the take-off and gives to growth an ongoing character". He further states that "the third condition implies a considerable capability to mobilize capital from domestic sources".

4. **The Drive to maturity:** This is the stage between take-off and maturity which is basically a long period of continued progress, with the level of investment rising as high as 20 percent of National Income. In general this stage

lasts 60 years. Rostow defined this "as the period when a society has effectively applied the range of modern technology to the bulk of its resources". The society thus reaches a stage in which its economy has both technological and entrepreneurial skills to produce not everything, but anything it wishes to produce.

5. **The Stage of High mass Consumption:** This stage is the longest. Rostow considers that it took the USA about 100 years to move from maturity to the level of this final stage. It is characterized by an affluent population and by mass production of sophisticated consumer goods and services. A close look at the theory will reveal that it has a) inadequate or poor empirical base; b) stages are not identifiable precisely; c) at no stage economic growth is automatic; c) misleading linear conception of history; e) hopeless attempt to refute Marx's theory of historical materialism; f) Rostow's argument has been overtaken by events in the LDCs.

7. Location theory

Location theory examines the forces which determine the location of economic activity and seeks to explain and predict the spatial pattern of the location of economic agents. Much of the theory has been concerned with the location decisions of producers or firms. Attention has also been given to the spatial pattern of agriculture, the distribution of towns and cities (particularly in urban economics) and to the location of households. A common tool used to analyze local economic strengths and weaknesses is the Location Quotient (LQ). A location quotient (LQ) is an analytical statistic that measures a region's industrial specialization relative to a larger geographic unit (usually the nation). LQ is computed as an industry's share of a regional total for some economic statistic (earnings, GDP by metropolitan area, employment, etc.) divided by the industry's share of the national total for the same statistic. For example, if the numerical value of LQ of 1.0 in mining it means that the region and the nation are equally specialized in mining. On the other hand, LQ of 1.8 means that the region has a higher concentration in mining than the nation. Using employment or income data for local sectors, this tool measures the ability of the local market to capture local economic activity. The Location Quotient's critical value is one, which is equal to the national average. If the computed LQ is greater than one for any sector, then that sector is considered to be a strength of that community; therefore, if the LQ is less than one, then the community is weak in that sector.

Location theory gives regional economics its scientific-disciplinary identity and constitutes its theoretical-methodological core. It has typically microeconomic foundations and it adopts a traditionally static approach. It deals with the location choices of firms and households. Linked with it are a variety of theoretical inputs (from macroeconomics, interregional trade theory, development theory, mathematical ecology, etc) which have refined the tools of regional economics and extended its range of inquiry. In microeconomic terms, location theory involves investigation into the location choices of firms and households. It also involves analysis of disparities in the spatial distribution of activities inquiry which enables interpretation of territorial disequilibria and hierarchies. Location theory uses the concepts of externalities and agglomeration economies to shed light on such macro-territorial phenomena as disparities in the spatial distribution of activities. Location theory seeks to explain the distribution of activities in space, the aim being to identify the factors that influence the location of individual activities, the allocation of different portions of territory among different types of production. Location models differ according to hypotheses on the spatial structure of demand and supply that reflect the aims that the models pursue. There are models whose aim is to interpret the location choices of firms. Choice of location is determined by an endeavour to minimize transportation costs between alternative locations and under the influence of agglomeration economies (theories of minimum-cost location). There are then models which seek to identify the market areas of firms, that is, the division of a spatial market among producers. Locational equilibrium is determined by a logic of profit maximization whereby each producer controls its own market area (theories of profit-maximizing location);

Weber's theory

Alfred Weber, a German economist, developed a systematic theory of industrial location in 1909. He analyzed the factors that determine the location of industry and classified these factors into two divisions:

(i) Primary causes of regional distribution of industry (regional factors); and (ii) Secondary causes (agglomerative and deglomerative factors) that are responsible for redistribution of industry.

(i) **Primary Causes: Regional Factors**

According to Weber, transport costs and labour costs are the two regional factors on which his pure theory is based. Assuming that there are no other factors that influence the distribution of industry, except transportation costs. Then it is clear that the location of industry will be pulled to those locations which have the lowest transportation costs. The key factors that determine transportation costs are(i) the weight to be transported; and (ii) the distance to be covered. Weber lists some more factors which influence the transportation costs such as (a) the type of transportation system and the extent of its use, (b) the nature of the region and kinds of roads, (c) the nature of goods themselves, i.e., the qualities which, besides weight, determine the facility of transportation. However, the location of the place of production must be determined in relation to the place of consumption and to the most advantageously located material deposits. Thus, 'locational figures' are created. These locational figures depend upon (a) the type of material deposits and (b) the nature of transformation into products. Weber classifies and calls those raw materials, which are available practically everywhere as 'ubiquities' (like brick-clay, water, etc) and 'localised' (like iron-ore, minerals, wood, etc) which are available only in certain regions. It is clear that localized materials play an important role on the industry than the ubiquities. Further, regarding the nature of the transformation of materials into products, Weber categorized the raw materials as 'pure' and 'weight losing'. Pure materials impart their total weight to the products (eg. cotton, wool, etc) and the materials are said to be 'weight losing' if only a part enters into the product (eg. wood, coal, etc.). Hence, the location of industries using weight-losing materials is drawn towards their deposits and that of industries using pure-materials towards the consumption centres. Weber further examines the cause of deviation of industrial location from the centres of least transport costs. The existence of differences in labour costs leads an industry to deviate from the optimal point of transport orientation. Geographical distribution of the population would give rise to differences in wages for labour. Naturally, the transport oriented location of an industry is drawn out and attracted towards the cheaper labour centres. Such migration of an industry from a point of minimum transport costs to a cheaper labour centre may be likely to occur only where the savings in the cost of labour are larger than the additional costs of transport which it ought to incur.

(ii) **Secondary Causes: Agglomerative and Deglomerative Factors**

Agglomeration happens when a large number of firms in a particular industry or market locate their headquarters, factories, and other facilities in one small geographic space. These clusters allow the firms to integrate their production externally with other firms. Deglomeration is the reverse process of this. An agglomerative factor is an advantage or a cheapening of production or marketing which results from the fact that production is carried on at one place. A deglomerative factor is a cheapening of production which results from the decentralization of production i.e., production in more than one place. To some extent these agglomerative and deglomerative factors also contribute to local accumulation and distribution of industry. These factors will operate only within the general framework formed by the two regional factors, i.e., costs of transportation and costs of labour. The advantages which could be derived in this context are external economies. The pulls which the agglomerative factors possess to attract an industry to a particular point are mainly dependent on two factors. Firstly, on 'the index of manufacture' (the proportion of manufacturing costs to the

total weight of the product) and secondly, on the 'locational weight' (the total weight to be transported during all the stages of production). To deduce a general principle, Weber uses the concept of "co-efficient of manufacture" which is the ratio of manufacturing cost to locational weight. Agglomeration is encouraged with high co-efficient of manufacture and deglomeration with low co-efficient of manufacture and these tendencies are inherent in their nature.

8. Theory of Innovation

In terms of regional economic development theory, Schumpeter is probably most well known for his theories linking economic development to innovation. He argues that traditional economic perspectives have been largely limited due to the inability of general equilibrium models to account for innovation. Instead, economic development follows a process that is largely path dependent and evolutionary. He describes the process of innovation as one of "creative destruction", where firms constantly search for new ways of doing business. He further argues that the process of innovation is largely a race for monopoly control over the stream of rents from new innovations. Technology strengthens innovation. Here, technology refers to the whole body of skills, knowledge and procedures for making, using and doing useful things. "Technology includes the nature and specification of what is produced as well as how it is produced. It encompasses managerial and marketing techniques as well as techniques directly involved in production. Technology extends to services administration, education, banking and the law, for example-as well as to manufacturing and agriculture. A complete description of the technology in use in a country would include the organization of productive units in terms of scale and ownership" (Francis Stewart, 1977). The development of techniques is essentially a historical process in which one technique with one set of characteristics replaces another in the light of the historical and economic circumstances of the time. At the same time, development as an ongoing process rests on the constant application of new technology and on the capacity to generate and absorb technical change. Innovation is used to cover both technological advances in production processes and the introduction of different attributes and attribute combinations in marketable products. Innovation is also a source of product differentiation and is used by producers to create demand and enhance market share. Most theories of economic development emphasize the need for technical progress. This progress usually requires capital, entrepreneurial skill, managerial and technical skills and a trained labour force which are, in general, scarce in LDCs. Inventions and innovations in one in one field stimulated inventions and innovations in other fields as well. Innovations in the post- industrial revolution western world were both cost-reducing and demand increasing. At the same time, the most damaging negative effect of technological innovations is that they lead to displacement of labour causing unemployment. Technical progress and innovations also bring about a profound change in the socio-economic environment. The standards of living increase, the basis for a fast growing urban and industrial system is laid, and boundless opportunities for progress unfold themselves. However, side by side, environmental pollution increases, anxiety and insecurity rises and old crafts and craftsmanship declines.

Today, technology has become an important determinant economic development and it is becoming more complex and involved. That is, man becomes a machine and gradually loses his identity in a fast changing world. Economists also argue that extensive reliance on less modern technology may divert needed energies and capital from the industrialization effort, thereby retarding ultimate development. However, there are also arguments for the selection of an appropriate technology in the place of neither capital intensive or labour intensive technologies. Technological progress may take three forms, viz. i) capital saving ii) labour saving, or iii) neutral. Given the rate of profit, technical change is said to be capital saving if it lowers the capital-output ratio, labour intensive if it raises the capital output ratio, neutral if it leaves the capital output ratio unchanged.

Three conditions are necessary for technical progress, viz, a) Technological progress requires large capital investment in research and development; and b) Dynamic entrepreneurs who have the ability and capacity to use scientific inventions and innovations; and c) There must be expanding mar-

ket for the product. For structural and for other reasons as well, technological progress is retarded in developing countries.

9. Marxist Theory

Marxist theory is basically structural in nature. The emergence of large metropolitan regions in the Western world particularly in the United States and France is the combined result of the monopolistic concentration of capital and the functional separation of large units of production. Underdeveloped regions have experienced fundamentally different growth histories than industrialized regions (Frank, 2004). The modern problem of underdevelopment can only be understood in terms of the historical development of the capitalist mode of production (Sweezy, 1946). In the Marxist literature the contradiction between the city and the country side are fundamentally contradictions between functional (interest based) forms of social integration and territorial (geographic and historical) forms of social integration. A proper understanding of dependency theory is the starting point of Marxist theory of development or regional development. That is, development, representing asymmetrical structural relations between social formations, such that the dependent society (ies) is shaped to a large extent by the social dynamics and interests generated in the dominant society (ies). However, Cardoso and Faletto (1979) argued that a simple core-periphery formulation could not account for the wide variation in the conditions of dependency even among Latin American societies. Instead, they called for an "historical-structural" approach that "emphasizes not just the structural conditioning of social life but also the historical transformations of structures by conflict, social movements, and class struggles".

10. Institutions and Development

Douglas North proposes a theory of economic development that is based on institutional adaptation and change. Operational flexibility and integrity of development largely depends on institutions. "Institutions are the humanly devised constraints that structure human interaction" including formal constraints and laws and informal constraints, such as norms, conventions and self-imposed codes of conduct (North, 1990). Organizations differ from institutions; "they are groups of individuals bound together by some common purpose to achieve certain objectives", and include legislatures, firms, trade unions, churches, clubs, schools, etc. Institutions are "the rules of the game in a society" while organizations are the players (North, 1990). But proper care in conceptualizing institutions creates problems in linking institutions with development. The essence of his argument is that political and economic institutions emerge primarily to resolve transaction cost dilemmas. The institutions that emerge establish the "rules of the game" for economic exchange and determine the expected private return from investments in the local economy. He elaborates on the significant role of institutions in economic development. There are four categories of institutions, viz, economic institutions, political institutions, legal institutions, and social institutions. Under economic institutions we include rules that define the production, allocation and distribution process of goods and services, including markets. Studies of political institutions usually employ variables that provide details about elections, electoral rules, type of political system, party composition of the opposition and the government, measures of checks and balances and political stability. Studies related to law and institutions refer to the type of legal systems, the definition and enforcement of property rights. Finally, studies on social institutions usually cover rules that have to do with access to health and education and social security arrangements, have an impact on gender balance and govern more generally the relationship between economic actors. Institutions serve to reduce the transaction costs associated with engaging in economic exchange by increasing the social benefits of long term co-operation. The institutions of capitalism have grown increasingly more complex due to the increasing complexity of economic exchanges. Institutional adaptation may either promote or discourage economic development. If the institutions that evolve are compatible with the transaction cost demands of private investors, then a region may not grow. For instance, if the property rights structure of a society does not recognize private contracts among economic agents, informal forms of governance may emerge to facilitate the capture of short term profits among es-

tablished elites while also serving to exclude outside investors. Similarly, large vertically integrated firms may emerge to monitor wage labour if contracts with external suppliers are not recognized.

11. Geography and Development

Krugman made an outstanding contribution to economic development by linking geography with trade. He suggests that recent models examining the role of geography in the economic development process have generally taken one of two approaches. The "predestination" approach explicitly considers the role of geographic factors determining the growth process. These geographic factors include climate and topography which determine patterns of regional growth and decline. The "self-organizing" approach asks why regional economies experience such different patterns of economic growth when there are no apparent geographic differences between regions. He concludes that why it is often difficult to draw policy conclusions from models linking geography and development. For new economic geographers, the primary difficulty lies in the fact that the models are highly stylized and typical versions of reality with outcomes that are very sensitive alternative assumptions and model specifications. Krugman also examines other models and emphasizes the importance of labour pooling and proximity to intermediate good suppliers on the emergence of regional clusters.

12. Conclusion

The theories, models and strategies of development, interalia, aim at identifying the key drivers of development. The key drivers in regional development largely depend on regional economic policy. Externalities, market failures, information asymmetries and co-ordination failures which in practice play an important role in shaping regional development trajectories are assumed not to exist. Their presence distorts the regional development system. It is argued that one should specify the regional parameters to be included in the spatial development policy. Conceptually, regional economic policy aims at altering the regional pattern of economic activity or economic performance. Regional policy also expresses solidarity with less developed regions, concentrating funds on the areas and sectors through broad investment policy in key strategic policy areas. It supports job creation, competitiveness, economic growth, improved quality of life, educational and health status and sustainable regional development. That is, spatial policy aims at reducing the significant economic, social and territorial disparities that exist between regions in the country.

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An Alternate Derivation for Thermodynamics of Quantum Gases

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Abstract

Thermodynamics of quantum gases are usually obtained using quantum statistical mechanics [1, 2, 3]. In this short communication we use the Mayer's cluster expansion technique [1, 3, 4] to reproduce the thermodynamics of free quantum systems. This method will help the students of statistical mechanics to have a visual picture about the interaction in quantum systems.

Keywords: Equation of state, statistical potential, Mayer cluster expansion.

1. Introduction

It is well known that for quantum systems like ideal bosons or fermions, even with out the external interacting potential, there is a statistical correlation between particles. This statistical correlation creates an attraction between bosons and repulsion between the fermions. This is responsible for the major differences in properties of these systems and also for the differences between the quantum and classical systems. One approach of explaining this correlation is by attributing symmetric wave functions for bosons and antisymmetric wave functions for fermions. Another way is by introducing a statistical inter particle potential given by [2, 3]

$$U_s(r) = -kT \ln[1 \pm e^{-\frac{2\pi r^2}{\lambda^2}}] \tag{1}$$

where r is the distance between the particles and λ is the thermal wavelength. This potential vanishes rapidly as $r > \lambda$. The studies on the nature of this potential shows it is attractive for bosons and repulsive for fermions [1, 2, 3, 5, 6] and this potential can be used to model the first order correction to the equation of state of ideal Bose and Fermi systems. The most accepted theory for deriving the thermodynamic properties of real systems is the Mayer's [4]theory of cluster expansion. The generalization of this theory which can be applied to gases obeying quantum statistics was developed by Kahn and Uhlenbeck and also by Lee and Yang [1, 3]. Earlier the cluster expansion method was used for finding the thermodynamics of the interacting system in the low density regime using pair wise potential models. The equation of states are derived for the potentials like hard core, Lennard Johnes, Sutherland and Cornell. Recently Ushcats [7] developed a new method to find the thermodynamics in the high density regime, beyond the radius of convergence of the virial expansions. He applied this for studying the critical behavior and phase transition by considering the Lennard Jones potential [7, 8]. A more accurate high density equation of state for lattice

gas model and also for Lee Yang lattice gas model were also developed based on this new formalism [9, 10]. Bannur [11] introduced a new generating function for canonical partition function which directly depends up on irreducible cluster integrals in order to study Mayer's convergence, virial expansion and condensation which was consistent with the Mayer's theory. Using this new generating function the equation of state in terms of irreducible cluster integral for finite N systems is also derived Ushcats [7]. Besides there is a great development in the cluster expansion technique for discussing the properties of quark gluon plasma and hadron phase transition [12, 13, 14, 15] by considering screened and unscreened color potentials like Cornell potential. The quark gluon plasma phase transition temperature obtained was in agreement with the predicted value based on the lattice calculations. All these studies show that cluster expansion method is a very effective technique in explaining the thermodynamics of real systems. So we revisit here the thermodynamics of quantum systems using the well known cluster expansion theory for bosons and fermions.

2. Theory

The Hamiltonian for an interacting system can be represented as

$$\sum_{i=1}^{N} \frac{p_i^2}{2m} + \sum_{i,j,i<j} U_{ij}(|\vec{r}_i - \vec{r}_j|) \tag{2}$$

where p is the momentum, m is the mass of particles, and U_{ij} is the potential that depends on the distance $|\vec{r}_i - \vec{r}_j|$ between the particles. The partition function with the above Hamiltonian is given by

$$Q_N(V,T) = \sum_{\{m_l\}} \left[\prod_{l=1}^{N} \left(\frac{b_l V}{\lambda^3} \right)^{m_l} \frac{1}{m_l!} \right] \tag{3}$$

Then using the cluster expansion theory, the equation of state of the system are

$$\frac{P}{kT} = \frac{1}{\lambda^3} \sum_l b_l z^l \tag{4}$$

$$\frac{N}{V} = \frac{1}{\lambda^3} \sum_l l b_l z^l \tag{5}$$

where P is the pressure, V is the volume, k the Boltzmann constant and $z = e^{\frac{\mu}{kT}}$ is the fugacity, with μ chemical potential of the system. N the total number of particles is given by $\sum_l l m_l = N$ where m_l is the number of clusters and b_l is the cluster integral given by

$$b_l(V,T) = \frac{1}{l!\lambda^{3(l-1)}V} \int_{V^l} \left[\sum \prod_{i,j \in \{l\}} f_{ij} \right] d^3 r_l \tag{6}$$

where $f_{ij} = e^{-\beta U_{ij}} - 1$ is the Mayer's function. The value of $b_1 = 1$ and $b_2 = \frac{2\pi}{\lambda^3} \int_0^\infty \left(e^{-\beta U_s(r)} - 1 \right) r^2 dr$ and so on. Using the free particle propagator theory [**?**] and the cluster expansion theory the values of the cluster integral for ideal collection of bosons and fermions in the thermodynamic limit is given by

$$b_l = \frac{(\pm 1)^{l-1}(l-1)!}{l!h^3} \int_0^\infty 4\pi p^2 e^{\frac{-l\beta p^2}{2m}} dp \tag{7}$$

Integrating we get

$$b_l = \frac{(\pm 1)^{l-1}}{l^{\frac{5}{2}}} \tag{8}$$

$(+)$ sign is for bosons and $(-)$ sign for fermions [1, 3]. Substituting

$$P = kT \frac{1}{\lambda^3} \sum_l \frac{(\pm 1)^{l-1} z^l}{l^{\frac{5}{2}}} \tag{9}$$

$$N = \frac{V}{\lambda^3} \sum_l \frac{(\pm 1)^{l-1} z^l}{l^{\frac{5}{2}}} \tag{10}$$

These equation of states are same as the equations derived using quantum statistical mechanics.

3. Conclusions

The thermodynamic behavior of a physical system can be deduced from the equation of state

and by using the statistical mechanics prescriptions. Starting from the statistical potential and using Mayer's cluster expansion we derived thermodynamics of ideal Bose and Fermi systems. The results completely matches with the calculations made by the quantum statistical method using the Bose and Fermi distribution functions. This shows that the concepts of Mayer's cluster expansion gives an interesting way for deriving the thermodynamic properties of the ideal quantum systems. This analysis gives some visual aspects for understanding a quantum system and its correlation which produce an effective attraction between bosons and repulsion between fermions with out the use of distribution functions.

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A Continuous Review Inventory Model for Slow Moving Items Under Fuzzy Demand

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Abstract

Slow moving inventory model with Laplace lead time demand is analyzed under fuzzy environments. Assuming the other components crisp, model under fuzzy annual demand is considered . The $(\rho, 1)$ interval valued fuzzy number is used to represent the fuzziness in the characteristic. An analytic solution procedure for deriving the optimum policy of the model is developed. Numerical illustrations are provided and comparison with the crisp case is performed.

Keywords: Inventory, Fuzzy set, Interval valued fuzzy number, Optimization.

1. Introduction

Many companies have to maintain slow moving products in inventory. What constitutes slow moving will vary from organization to organization .The items such as carpets, mats, sports goods like chess board, carron board etc, ventilators, certain spare parts of machines and vehicles, etc are generally considered to be slow moving items. The most prevalent variations due to risk and uncertainties in an inventory system are variations in lead time and demand. The provision for safety stocks, also referred to as buffer stocks is intended to overcome such variations. Safety stocks are extra inventory kept on hand against stockouts due to unforseen events. As such, the knowledge of the distribution of lead time demand is essential for determining the inventory decision variables. For slow moving items, usually the lead time demand tends to be positively skewed. Laplace distributions are regarded as ideal to model lead time demand for slow moving items [Presutti and Trepp (1972), Archibad *et al.*(1974), Peterson and Silver (1979)]. Bagchi *et al.* (1983) proposed Hermite distribution while Agarwal and Smith (1996) found the negative binomial distribution to describe lead time demand of slow moving items. Bagchi (1987) developed a slow moving inventory model where the arrival of customer orders follows a Poisson process and the order sizes are geometrically distributed. Recently, Chang *et al.* (2001) developed a continuous review slow moving inventory model with Laplace lead time demand.

In most of the stochastic inventory models, though the lead time or the total demand is random, the expected demand per year (annual demand) is taken as a fixed value. Since the demand characteristic is highly subjective and linguistic (based on the users preference) in nature, fuzzy set is a suitable way to model this vague characteristic. Several authors have applied the fuzzy set concepts to deal with the

inventory control problems. Park (1987) examined the Economic Order Quantity (EOQ) model from the fuzzy set theoretic perspective by using trapezoidal fuzzy numbers for ordering and inventory holding costs in the model. Ishii and Konno (1998) fuzzified the shortage cost to an L-fuzzy number in the classical newsboy problem. A backorder inventory model which fuzzifies the order quantity as triangular and trapezoidal fuzzy numbers and keeps the shortage cost as a crisp parameter was developed by Yao and Lee (1999). Yao and Su (2000) considered fuzzy inventory model without backorder by adopting an interval valued fuzzy number for fuzzifying the total demand quantity. Quyang and Yao (2002) applied the minimax decision approach to solve a continuous review mixture inventory model in which the annual demand is fuzzy. Kao and Hsu (2002) studied a continuous review model where fuzziness is introduced in the demand. Pai and Hsu (2003) studied continuous review inventory problem in a fuzzy environment where the demand and holding costs are represented by fuzzy sets. Pan and Yang (2006) analyzed continuous review mixture model in fuzzy demand with controllable lead time. A mixture inventory model involving variable lead time with backorders and lost sales where the random lead time is fuzzified by a random fuzzy variable and the total demand by trapezoidal fuzzy numbers was developed by Chang et al. (2006). A fuzzy stochastic continuous review model where the average storage space and total expenditure are fuzzified by linear membership functions was discussed by Das et al. (2006). A continuous review inventory system where fuzziness and randomness appear simultaneously into an optimization setting was discussed by Dutta et al. (2007). Vijayan and Kumaran (2008) described both periodic and continuous review inventory model with a mixture of backorders and lost sales where the cost components in the models are represented by fuzzy sets.

In this paper a continuous review (Q, r) inventory model for slow moving items with Laplace lead time demand is analyzed under fuzzy annual demand. This paper is organized as follows. Section 2 introduces the (Q, r) inventory model for slow moving items. Section 3 reviews the basic concepts of fuzzy set theory. The model under fuzzy annual demand is described in section 4. The signed distance method is adopted to defuzzify the fuzzy cost function. An analytic procedure for determining the optimum policy of the model is derived. Results of numerical computations for optimum parameters of the model under both fuzzy and crisp cases and their comparisons are presented in section 5. Finally, the conclusion is given in the last section.

2. Continuous review (Q, r) inventory model for slow moving items

(Q, r) inventory model or the reorder point model is a continuous review inventory system in which the inventory position of an item is monitored after every transaction and the policy is to order a lot of size Q units when the inventory level drops to the reorder point r. Such models are described in all standard text books on inventory. In order to describe one such model for slow moving items, the following assumptions and notations are introduced.

Assumptions

(i) A single item is considered.

(ii) Average annual demand is fixed.

(iii) If two or more replenishment orders for the same item are simultaneously outstanding, then, they are received in the same order in which they are placed.

(iv) The average level of backorders is negligibly small when compared with the average level of the on-hand stock.

(v) Lead time demand Y follows Laplace distribution with mean θ and standard deviation σ.

(vi) The costs of control system do not depend on the specific value of the reorder point.

Notations

D = average annual demand.
τ = replenishment lead time, in years.
C = ordering cost per inventory cycle.
k = safety factor, $k \geq 0$, safety stock= $k\sigma$ units.
h = inventory carrying charge per year.
c_1 = unit variable cost per unit.
c_0 = specified fractional charge per unit short.
Reorder point = $r = \theta + k\sigma$.

The expression for expected total cost in Chang *et al.* (2001) is given by

$$Z(k,Q) = \frac{CD}{Q} + \left(\frac{Q}{2} + k\sigma\right) hc_1 + \frac{c_1 D c_0\ \sigma\ e^{-\sqrt{2k}}}{2\sqrt{2}\ Q}. \tag{1}$$

The function $Z(k,Q)$ is convex on $k \geq 0$ and $Q > 0$ [(Chang *et al.* (2001)]. Hence the optimum values of Q and k are derived by the usual calculus procedure of minimization as

$$Q = Q(k) = \sqrt{\frac{(2\sqrt{2CD} + c_1 c_0 \sigma D)e^{-\sqrt{2k}}}{\sqrt{2}hc_1}}, \tag{2}$$

and

$$k = \frac{1}{\sqrt{2}}\ ln\left(\frac{c_0 D}{2Qh}\right). \tag{3}$$

Eqs. (2) and (3) are not explicit to find the values of Q and k. Chang and Huang (2003) proposed an analytic procedure to find the optimum values of Q and k, as given below.
Let

$$\Delta = \frac{-\sigma}{h} + \frac{c_0 D}{\sqrt{2h^2}}, \tag{4}$$

$$\varphi(k) = Q(k)^2 - \frac{(c_0 D)^2 e^{-2\sqrt{2k}}}{4h^2}, \tag{5}$$

$$\psi(k) = \frac{-\sigma}{h} + \frac{c_0 D e^{-\sqrt{2k}}}{\sqrt{2h^2}}. \tag{6}$$

(i) If $\Delta \leq 0$, then the optimum values of k and Q are respectively $k = 0$ and $Q = Q(0)$.

(ii) For $\Delta > 0$ and $\varphi(0) \geq 0$, the optimum solution is $k = 0$ and $Q = Q(0)$.

(iii) In cases $\Delta > 0$ and $\varphi(0) < 0$, then there exist a number k^* satisfying $0 < k^* < k^{'}$ such that $\varphi(k^*) = 0$ and $\psi(k^{'}) = 0$. In such case, the optimum solution is $k = k^*$ and $Q = Q(k^*)$.

Some preliminary concepts of fuzzy set theory required in the development of our model are described below.

3. Fuzzy Set

In a universe of discourse X, a fuzzy subset $\tilde{A}$ on X is defined by the membership function $\mu_{\tilde{A}}(x)$ which maps each element x in X to a real number in the interval $[0,1]$. $\mu_{\tilde{A}}(x)$ denotes the grade or degree of membership and it is usually denoted as $\mu_{\tilde{A}} : X \to [0,1]$. A fuzzy set is said to be normal if

the largest grade obtained by any element in that set is 1. That is, there must exist at least one x for which $\mu_{\tilde{A}}(x) = 1$. A fuzzy set $\tilde{A}$ on X is convex iff $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq min\ (\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$ for all $x_1, x_2 \varepsilon\ X$ and for $\lambda \varepsilon\ [0,1]$, where *min* denotes the minimum operator.

Definition 1: *Fuzzy number*

A fuzzy number is a fuzzy subset of the real line which is both normal and convex. In addition, the membership function of a fuzzy number must be piecewise continuous.
The membership function of a fuzzy number $\tilde{A}$ is usually represented as

$$\begin{aligned} \mu_{\tilde{A}}(x) &= l(x),\ x < m, \\ &= 1,\ m \leq x \leq n, \\ &= u(x),\ x > n, \end{aligned} \tag{7}$$

where $l(x)$ is continuous from the right, strictly increasing for $x < m$ and there exist $m_1 < m$ such that $l(x) = 0$ for $x \leq m_1$ and $u(x)$ is continuous from the left, strictly decreasing for $x > n$ and there exist $n_1 \geq n$ such that $u(x) = 0$ for $x \geq n_1$. $l(x)$ and $u(x)$ are called the left and right reference functions respectively.

Fuzzy sets defined by membership functions in a closed interval of real numbers between the identified lower and upper bounds are called interval valued fuzzy sets. These sets are defined formally by functions of the form $\mu_{\tilde{A}} : X \to I([0,1])$, where $I([0,1])$ denotes the family of all closed intervals of real numbers in $[0,1]$. Interval-valued fuzzy sets provide a more adequate description of uncertainty than traditional fuzzy sets.

Definition 2: *Interval valued fuzzy set.*

A fuzzy set $\tilde{A}$ defined on the set of real numbers is called an interval valued fuzzy set if its membership function is given by

$$\tilde{A} = \{\big(x, [\mu_{\tilde{A}^L}(x), \mu_{\tilde{A}^U}(x)]\big)\},\ 0 \leq \mu_{\tilde{A}^L}(x) \leq \mu_{\tilde{A}^U}(x) \leq 1.$$

Symbolically, interval valued fuzzy set $\tilde{A}$ is denoted by $[\tilde{A}^L, \tilde{A}^U]$. Obviously the largest grade of x in $\tilde{A}$ is $\mu_{\tilde{A}^U}(x)$ and smallest grade is $\mu_{\tilde{A}^L}(x)$.

Definition 3: *Interval valued fuzzy number.*

The fuzzy number $\tilde{A} = (a_1, a_2, a_3; \rho)$ is a level ρ fuzzy number, if $0 < \rho \leq 1$, $a_1 < a_2 < a_3$, and $\tilde{A}$ is a fuzzy set on the set of real numbers with membership function,

$$\begin{aligned} \mu_{\tilde{A}}(x) &= \frac{\rho\,(x - a_1)}{a_2 - a_1},\ a_1 \leq x \leq a_2, \\ &= \frac{\rho\,(x - a_3)}{a_2 - a_3},\ a_2 \leq x \leq a_3, \\ &= 0,\ \text{otherwise}. \end{aligned} \tag{8}$$

In particular if $\tilde{A}^L = (a_1, a_2, a_3;\ \rho_1)$ and $\tilde{A}^U = (a_0, a_2, b_0;\ \rho_2)$, the fuzzy number $\tilde{A} = [\tilde{A}^L, \tilde{A}^U] = [(a_1, a_2, a_3;\ \rho_1), (a_0, a_2, b_0;\ \rho_2)]$, $a_0 < a_1 < a_2 < a_3 < b_0$ is a level $(\rho_1,\ \rho_2)$ interval valued fuzzy number.

The membership function of $\tilde{A} = [\tilde{A}^L,\ \tilde{A}^U]$ is defined as

$$\begin{aligned}\mu_{\tilde{A}^L}(x) &= \frac{\rho_1\ (x - a_1)}{a_2 - a_1},\ a_1 \le x \le a_2,\\ &= \frac{\rho_1\ (x - a_3)}{a_2 - a_3},\ a_2 \le x \le a_3,\\ &= 0,\ \text{otherwise},\end{aligned} \tag{9}$$

and

$$\begin{aligned}\mu_{\tilde{A}^U}(x) &= \frac{\rho_2\ (x - a_0)}{a_2 - a_0},\ a_0 \le x \le a_2,\\ &= \frac{\rho_2\ (x - b_0)}{a_2 - b_0},\ a_2 \le x \le b_0,\\ &= 0,\ \text{otherwise}.\end{aligned} \tag{10}$$

Signed distance method

In order to defuzzify the fuzzy cost function in section 4, we need to consider some distance measures in Yao and Wu (2000), Yao and Lin (2003) and Chiang (2005).

The signed distance from the real number a to 0 is defined as $d_0(a, 0) = a$. If $a > 0$, $d_0(a, 0)$ is the distance from a to 0. If $a < 0$, the distance from a to 0 is taken such that $-d_0(a, 0) = -a$.

For any closed interval $[a, b]$, the signed distance of $[a, b]$ measured from 0 is defined as $d_0\left([a, b], 0\right) = \frac{1}{2}(a + b)$. For any two disjoint closed intervals $[a, b]$ and $[c, d]$, the signed distance of $[a, b] \cup [c, d]$ from 0 is defined as

$$d_0\left([a, b] \cup [c, d],\ 0\right) = \frac{1}{2}\left(d_0([a, b], 0) + d_0([c, d], 0)\right) = \frac{1}{4}(a + b + c + d).$$

Let $\tilde{A} = [(a_1, a_2, a_3; \rho_1),\ (a_0, a_2, b_0;\ \rho_2)],\ a_0 < a_1 < a_2 < a_3 < b_0$ be a level $(\rho_1,\ \rho_2)$ interval valued fuzzy number. The signed distance of its α cuts to the one level fuzzy point $\tilde{0}$ for $0 \le \alpha \le \rho_1$ and $\rho_1 \le \alpha \le \rho_2$ are given below.

The signed distance of $\tilde{A}$ to $\tilde{0}$ when $0 \le \alpha \le \rho_1$ is

$$\begin{aligned}&d\left(\tilde{A}_{[0,\ \rho_1]}(\alpha);\ \tilde{0}\right)\\ &= \frac{1}{\rho_1}\int_0^{\rho_1}\frac{1}{4}\left(a_0 + a_1 + a_3 + b_0 + (2a_2 - a_1 - a_3)\frac{\alpha}{\rho_1} + (2a_2 - a_0 - b_0)\frac{\alpha}{\rho_2}\right)\ d\alpha.\end{aligned} \tag{11}$$

Similarly, when $\rho_1 \le \alpha \le \rho_2$, the signed distance from $\tilde{A}$ to $\tilde{0}$ is

$$d\left(\tilde{A}_{[\rho_1,\ \rho_2]}(\alpha);\ \tilde{0}\right) = \frac{1}{\rho_2 - \rho_1}\int_{\rho_1}^{\rho_2}\frac{1}{2}\left(a_0 + b_0 + (2a_2 - a_0 - b_0)\frac{\alpha}{\rho_2}\right)\ d\alpha. \tag{12}$$

The signed distance of the level $(\rho_1,\ \rho_2)$ interval value fuzzy number $\tilde{A}$ to $\tilde{0}$ is defined as the mean value on $[0, \rho_2]$ through integration. That is,

$$d(\tilde{A},\ \tilde{0}) = \frac{1}{\rho_2}\ \left(\rho_1\ d\left(\tilde{A}_{\ [0,\ \rho_1]}(\alpha);\ \tilde{0}\right) + (\rho_2 - \rho_1)\ d\left(\tilde{A}_{\ [\rho_1,\ \rho_2]}(\alpha);\ \tilde{0}\right)\right). \tag{13}$$

The signed distance formula in eq. (13) is considered for defuzzification problems and when ranking fuzzy numbers.

Let $\tilde{A}_1$ and $\tilde{A}_2$ are two (ρ_1, ρ_2) interval valued fuzzy numbers, then the following properties are satisfied.

(i) $d(\tilde{A_1} + \tilde{A_2}, \tilde{0}) = d(\tilde{A_1}, \tilde{0}) + d(\tilde{A_2}, \tilde{0})$.

(ii) If $k > 0$, then $d(k\tilde{A_1}, \tilde{0}) = kd(\tilde{A_1}, \tilde{0})$.

(iii) d is a linear operator.

(iv) $\tilde{A_1} < \tilde{A_2}$ if $d(\tilde{A_1}, \tilde{0}) < d(\tilde{A_2}, \tilde{0})$.

4. (Q, r) model for slow moving items under fuzzy annual demand

In this section, model in section 2 is considered under fuzzy annual demand. Assuming all other components in the model given by eq. (1) as crisp, the annual demand D represented by $(\rho, 1)$ interval valued fuzzy number as $\tilde{D} = [(D - \eta_1, D, D + \eta_2;\ \rho),\ (D - \eta_3, D, D + \eta_4;\ 1)]$ where $\eta_i, i = 1, 2, 3, 4$ are arbitrary positive constants which satisfy $\eta_1 < \eta_3$ and $\eta_2 < \eta_4$
The signed distance from $\tilde{D}$ to $\tilde{0}$ when $0 \leq \alpha \leq \rho$ and $\rho \leq \alpha \leq 1$ are respectively given from eqs. (11) and (12) by

$$d\left(\tilde{D}_{\ [0,\ \rho]}\ (\alpha);\ \tilde{0}\right) = \frac{1}{8}\left(8D - \eta_1 + \eta_2 - 2\eta_3 + 2\eta_4 + (\eta_3 - \eta_4)\rho\right). \tag{14}$$

and

$$d\left(\tilde{D}_{\ [\rho,\ 1]}\ (\alpha);\ \tilde{0}\right) == \frac{1}{4}\left(4D - 2\eta_3 + 2\eta_4 + (\eta_3 - \eta_4)(1 + \rho)\right). \tag{15}$$

Using eqs. (13)- (15), the signed distance from $\tilde{D}$ to $\tilde{0}$ is

$$\begin{aligned} d(\tilde{D}, \tilde{0}) = &\ \rho\, d\left(\tilde{D}_{\ [0,\ \rho]}\ (\alpha);\ \tilde{0}\right) + (1 - \rho)\, d\left(\tilde{D}_{\ [\rho,\ 1]}\ (\alpha);\ \tilde{0}\right), \\ = &\ \frac{\rho}{8}(\eta_2 - \eta_1 + 2\eta_3 - 2\eta_4) + \frac{1}{4}(4D + \eta_4 - \eta_3) + \frac{\rho^2}{8}(\eta_4 - \eta_3). \end{aligned} \tag{16}$$

From eq. (1), the cost function under fuzzy annual demand is given by

$$Z(k, Q, \tilde{D}) = \frac{C\tilde{D}}{Q} + \left(\frac{Q}{2} + k\sigma\right) hc_1 + \frac{c_1 c_0 \tilde{D}\sigma e^{-\sqrt{2k}}}{2\sqrt{2Q}} \tag{17}$$

Since d is a linear operator, the signed distance from $Z(k, Q, \tilde{D})$ to $\tilde{0}$ is

$$\begin{aligned} d\left(Z(k, Q, \tilde{D}), \tilde{0}\right) &= d\left(\frac{C\tilde{D}}{Q} + \left(\frac{Q}{2} + k\sigma\right) hc_1 + \frac{c_1 c_0 \tilde{D}\sigma e^{-\sqrt{2k}}}{2\sqrt{2Q}},\ \tilde{0}\right) \\ &= \frac{C}{Q}\, d(\tilde{D}, \tilde{0}) + \left(\frac{Q}{2} + k\sigma\right) hc_1 + \frac{c_1 c_0 \sigma e^{-\sqrt{2k}}}{2\sqrt{2}\ Q}\, d(\tilde{D},\ \tilde{0}) \end{aligned} \tag{18}$$

The signed distance $d\big(Z(k, Q, \tilde{D}), \tilde{0}\big)$ denoted by $E(Z, D)$ is the crisp estimate of fuzzified function in eq. (17). $E(Z, D)$ is a convex function of k and Q (similar to eq. (1)), and the values of k and Q which minimize eq. (18) are obtained by solving

$$\frac{\partial E(Z, D)}{\partial k} = 0 \tag{19}$$

and

$$\frac{\partial E(Z, D)}{\partial Q} = 0. \tag{20}$$

The above two equations reduce to

$$\sigma h c_1 - \frac{c_1 c_0\ d(\tilde{D},\tilde{0})\ \sigma\ \exp(-\sqrt{2k})}{2Q} = 0, \tag{21}$$

$$\frac{-C\ d(\tilde{D},\tilde{0})}{Q^2} + \frac{h c_1}{2} - \frac{c_1 c_0\ d(\tilde{D},\tilde{0})\ \sigma\ \exp(-\sqrt{2k})}{2\sqrt{2}Q^2} = 0, \tag{22}$$

Solving eqs. (21) and (22), we get

$$Q = Q_{\tilde{D}}(k) = \sqrt{\frac{\left(2\sqrt{2}C\ d(\tilde{D},\tilde{0})\ + c_1 c_0 \sigma\ d(\tilde{D},\tilde{0})\ e^{-\sqrt{2k}}\right)}{\sqrt{2}hc_1}}, \tag{23}$$

and

$$k = k_{\tilde{D}} = \frac{1}{\sqrt{2}}\ ln\left(\frac{c_0\ d(\tilde{D},\tilde{0})}{2Qh}\right). \tag{24}$$

The above notations $Q_{\tilde{D}}(k)$ and $k_{\tilde{D}}$ respectively denote the optimum values of Q and k under fuzzy annual demand. Eqs. (23) and (24) are not directly solvable for the values of Q and r. In fact, for $k \geq 0$, eq. (24) will have no solution if $\frac{c_0\ d(\tilde{D},\tilde{0})}{2Qh} < 1$. Let us consider the simultaneous solution set for eqs. (23) and (24),

$$\begin{aligned}\varphi_{\tilde{D}}(k) &= \frac{2(2\sqrt{2}C\ d(\tilde{D},\tilde{0})\ + c_1 c_0 \sigma\ d(\tilde{D},\tilde{0})\ e^{-\sqrt{2k}})}{2\sqrt{2}hc_1} - \frac{(c_0\ d(\tilde{D},\tilde{0})\ )^2 e^{-2\sqrt{2}k}}{4h^2}, \\ &= Q_{\tilde{D}}(k)^2 - \frac{(c_0\ d(\tilde{D},\tilde{0})\ )^2 e^{-2\sqrt{2}k}}{4h^2}.\end{aligned} \tag{25}$$

Define

$$\psi_{\tilde{D}}(k) = \frac{-\sigma}{h} + \frac{c_0\ d(\tilde{D},\tilde{0})\ e^{-\sqrt{2}k}}{\sqrt{2}h^2}, \tag{26}$$

and

$$\Delta_{\tilde{D}} = \psi_{\tilde{D}}(0) = \frac{-\sigma}{h} + \frac{c_0\ d(\tilde{D},\tilde{0})}{\sqrt{2}h^2}. \tag{27}$$

Eqs. (25)-(27) are similar to eqs. (4) - (6). Hence the solution procedure of the model can be described by the following algorithm.

Step(i) If $\Delta_{\tilde{D}} \leq 0$, set $k_{\tilde{D}} = 0$ and go to step (iv). Otherwise go to step(ii).

Step(ii) If $\varphi_{\tilde{D}}(0) \geq 0$, set $k_{\tilde{D}} = 0$, go to step (iv). Otherwise go to step (iii).

Step(iii) Find two arbitrary positive real numbers k^* and $k^{'}$, satisfying $0 < k^* < k'$ such that $\psi_{\tilde{D}}(k^{'}) = 0$ and $\varphi_{\tilde{D}}(k^*) = 0$. Set $k_{\tilde{D}} = k^*$ and go to step (iv)

Step(iv) Find the optimal order quantity $Q_{\tilde{D}}$ and the optimum reorder point $r_{\tilde{D}} = d(\tilde{D},\tilde{0})\ + k_{\tilde{D}}\ \sigma$.

5. Numerical study

Numerical illustrations of the models developed under fuzzy cases in section 4 and crisp case in section 2 are made. The sensitiveness in the optimum decision variables and optimum total cost under fuzziness are studied.

Crisp cases

We consider a continuous review inventory system with crisp characteristics chosen in Chung and Huang (2003). Depending on the values of Δ, φ and ψ, in eqs. (4) to (6), the optimum policy corresponding to three cases arise.

(i)$\Delta \leq 0$, (ii) $\Delta > 0$ and $\varphi(0) \geq 0$ and (iii) $\Delta > 0$ and $\varphi(0) < 0$

Case(i) $\Delta \leq 0$

$D = 80$ units/year, $C = \$30$, $\tau = 2$ weeks $= 2/52$ years, $\sigma = 1$unit, $h = \$0.4$ /unit/year, $c_1 = \$350$/unit, $c_0 = \$0.005$.

Then, $\Delta = -0.7322 < 0$, $Q = 5.9155$, $k = 0$, $r = D\tau + k\sigma = 3.0769$ and $Z(Q, k) = \$828.1662$.

Case (ii) $\Delta > 0$ **and** $\varphi(0) \geq 0$

$D = 800$ units/year, $C = \$40$, $\tau = 2$ weeks $= 2/52$ years, $\sigma = 5$ units, $h = \$0.24$ /unit/year, $c_1 = \$2$/unit, $c_0 = \$0.2$.

$\Delta = 1943.4 > 0$ and the optimum solution is $Q = 368.3617$, $k = 0$, $r = 30.7692$ and $Z(Q, k) = \$176.82$.

Case (iii) $\Delta > 0$ **and** $\varphi(0) < 0$

$D = 104$ units/year, $C = \$20$, $\tau = 2$ weeks $= 2/52$ years, $\sigma = 1$ units, $h = \$0.24$ /unit/year, $c = \$350$/unit, $c_0 = \$0.2$. In such case $\Delta = 251.1775 > 0$ and the optimum values $Q = 7.7798$, $k = 1.2144$, $r = 5.2144$ and $Z(Q, k) = \$755.52$.

Fuzzy cases

(Q, r) *inventory model for slow moving items under fuzzy annual demand*

Like crisp cases, here also depending on the values of $\Delta_{\widetilde{D}}, \varphi_{\widetilde{D}}$ and $\psi_{\widetilde{D}}$, optimum policy corresponding to three cases arise.

(i)$\Delta_{\widetilde{D}} \leq 0$, (ii) $\Delta_{\widetilde{D}} > 0$ and $\varphi_{\widetilde{D}}(0) \geq 0$ and (iii) $\Delta_{\widetilde{D}} > 0$ and $\varphi_{\widetilde{D}}(0) < 0$

Case (i) $\Delta_{\widetilde{D}} \leq 0$

Here we consider the continuous review inventory system with crisp characteristics as mentioned above in case(i) of crisp cases. The crisp value of annual demand is $D = 80$. In order to fuzzify the annual demand D, we choose the following $(\rho, 1) = (.8, 1)$ interval valued fuzzy numbers in the ranking order as per eq. (13) by arbitrary choices of $\eta_i, i = 1, 2, 3, 4$.. The corresponding defuzzified value (singed distance) and the percentage increase in the fuzzy case from the crisp case (denoted by P_D) and $\Delta_{\widetilde{D}}$ are given below. It should be noted that the entries in the third row (all the components of $\tilde{D}$ are equal) is equivalent to the crisp.

$\tilde{D}$	$d(\tilde{D}, \tilde{0})$	P_D	$\Delta_{\widetilde{D}}$
$[(10, 80, 81; .8)\ (5, 80, 85; 1)]$	64	-20	-1.0858
$[(50, 80, 85; .8)\ (25, 80, 93; 1)]$	72	-10	-0.9090
$[(80, 80, 80; .8)\ (80, 80, 80; 1)]$	80	0	-0.7322
$[(70, 80, 100; .8)\ (50, 80, 164; 1)]$	88	+10	-0.5555
$[(70, 80, 120; .8)\ (60, 80, 200; 1)]$	96	+20	-0.3787

Table 1. *Optimum policy under fuzzy annual demand when* $\Delta_{\widetilde{D}} < 0$.

P_D	Q	% change in Q	r	% change in r	$E(Z,\tilde{D})$	% change in $E(Z,\tilde{D})$
-20	5.2910	10.6	2.4615	20	740.7344	10.6
-10	5.6119	5.12	2.7692	10	785.6675	5.12
0	5.9155	0.00	3.0769	0	828.1662	0.00
$+10$	6.2042	4.88	3.3846	10	868.5881	4.88
$+20$	6.4801	9.55	3.6923	20	907.2106	9.55

Case (ii) $\Delta_{\widetilde{D}} > 0$ **and** $\varphi_{\widetilde{D}}(0) \geq 0$

The inventory system with crisp components chosen above in case (ii) of crisp cases is considered where the annual demand $D = 800$. The following $(\rho, 1) = (0.8, 1)$ interval valued fuzzy numbers and the corresponding values of $d(\tilde{D}, \tilde{0}), P_D$, $\Delta_{\widetilde{D}}$ and $\varphi_{\widetilde{D}}(0)$ are given below.

$\tilde{D}$	$d(\tilde{D},\tilde{0})$	P_D	$\Delta_{\widetilde{D}}$	$\varphi_{\widetilde{D}}(0)$
$[(100, 800, 810; .8)\ (50, 800, 850; 1)]$	640	-20	1550.5	37441
$[(500, 800, 850; .8)\ (250, 800, 930; 1)]$	720	-10	1746.9	32121
$[(800, 800, 800; .8)\ (800, 800, 800; 1)]$	800	0	1943.4	24579
$[(700, 800, 1000; .8)\ (500, 800, 1640; 1)]$	880	$+10$	2139.8	14815
$[(700, 800, 1200; .8)\ (600, 800, 2000; 1)]$	960	$+20$	2336.2	2828

Table 2. *Optimum policy under fuzzy annual demand when* $\Delta_{\widetilde{D}} > 0$ *and* $\varphi_{\widetilde{D}}(0) \geq 0$.

P_D	Q	% change in Q	r	% change in r	$E(Z,\tilde{D})$	% change in $E(Z,\tilde{D})$
-20	329.4727	10.6	24.6154	20	158.1469	10.6
-10	349.4586	5.13	27.6923	10	167.7401	5.13
0	368.3617	0.00	30.7692	00	176.8136	0.00
$+10$	386.3410	4.88	33.8462	10	185.4437	4.88
$+20$	403.5200	9.54	36.9231	20	193.6896	9.54

Case (iii) $\Delta_{\widetilde{D}} > 0$ **and** $\varphi_{\widetilde{D}}(0) < 0$

Assuming the crisp characteristics in case (iii) of crisp cases with annual demand $D = 104$, the following $(\rho, 1) = (.8, 1)$ interval valued fuzzy number with corresponding signed distance, percentage increase in fuzzy case from the crisp case, $\Delta_{\widetilde{D}}$, $\varphi_{\widetilde{D}}(0)$ and k' are given below.

$\tilde{D}$	$d(\tilde{D},\tilde{0})$	P_D	$\Delta_{\widetilde{D}}$	$\varphi_{\widetilde{D}}(0)$	k'
$[(12, 104, 105; .8)\ (10, 104, 108; 1)]$	83.20	-20	200.109	-1113	2.7512
$[(50, 104, 122; .8)\ (25, 104, 130; 1)]$	93.60	-10	225.643	-1421	2.8356
$[(104, 104, 104; .8)\ (104, 104, 104; 1)]$	104.0	0	251.178	-1767	2.9101
$[(81, 104, 150; .8)\ (70, 104, 200; 1)]$	114.4	$+10$	276.712	-215	2.9772
$[(95, 104, 175; .8)\ (80, 104, 240; 1)]$	124.8	$+20$	302.246	-257	3.0388

Table 3. *Optimum policy under fuzzy annual demand when* $\Delta_{\widetilde{D}} > 0$ *and* $\varphi_{\widetilde{D}}(0) < 0$.

P_D	k	Q	% change in Q	r	% change in r	$E(Z,\tilde{D})$	% change in $E(Z,\tilde{D})$
-20	1.12719	7.0410	9.50	4.3272	17.0	686.13	9.18
-10	1.17331	7.4206	4.62	4.7733	8.46	721.89	4.45
0	1.21440	7.7798	0.00	5.2144	0.00	755.52	0.00
$+10$	1.26364	8.1095	4.24	5.6636	8.61	787.35	4.21
$+20$	1.30117	8.4325	8.39	6.1012	17.0	817.62	8.22

Tables 1 to 3 show variations in the optimum order quantity Q and reorder point r under fuzzy annual demand. Increase in fuzzy annual demand (in terms of fuzzy ranking) causes considerable increases in Q, r and the optimum total cost. It should be noted that while the optimum values of Q and r and the total cost differ widely, the optimal values of r registers a percentage change more or less equal to those in D due to fuzziness in the annual demand. The percentage changes in Q and the total cost influence approximately half of the percentage change in D at each level of fuzziness.

6. Conclusion

(Q, r) inventory model for slow moving items is developed under fuzzy annual demand. The fuzziness in the annual demand is represented by the $(\rho, 1)$ interval valued fuzzy number. We have adopted the signed distance method to defuzzify the fuzzy cost function of the model. The study reveals that the optimum order quantity and the total cost are sensitive due to fuzziness in the annual demand. The changes in the optimum order quantity and the total cost is nearly half of the percentage change due to fuzziness in the annual demand. The optimum reorder level is highly sensitive under fuzzy annual demand. The decision maker should adopt a better trade of judgement for accounting flexibility in the characteristics of the model in order to tackle the uncertainty which fits to the real situations.

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Preparation and Characterization of Lead Sulphide Thin Films

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Abstract

PbS thin films were deposited on properly cleaned glass and polyethylene terepthalate (PET) substrates by chemical bath deposition (CBD) technique at room temperature using Lead Acetate Trihydrate $Pb(CH_3COO)_2.3H_2O$, Thioacetamide C_2H_5NS, Triethanolamine (TEA) $C_6H_{15}NO_3$. The structural properties were determined by X-ray diffraction studies. The surface morphology was determined by scanning electron microscopy. The optical properties were carried out from spectroscopy measurements of absorption-transmissionreflection

1. Experimental

PbS films were grown by chemical bath deposition at room temperature, where the substrates were immersed inclined to the beaker. A stock solution of 100 ml was prepared using 15 ml 0.6 M of lead acetatetrihydrate $Pb(CH_3COO)_2.3H_2O$ as source of Pb+ ions, 15 ml 0.6 M of Thioacetamide C_2H_5NS to produce S- ions, 15ml 1M of Triethanolamine (TEA) $C_6H_{15}NO_3$ as complexing agent and Distilled water. The chemical bath is kept for 3 hour. After the film formed the film coated substrates were removed and rinse with distilled water then dried by using hot air blower. The mirror- like films formed was smooth and patch free and brown in colour. The films showed good adherence. As-grown films coated on glass substrates were named as CPBS1 and that coated on polyethylene terephthalate as CPBS1-PET.

2. Results and Discussions

2.1. Thickness of the films

Thickness of PbS film was computed by gravimetric method. The estimated thickness is depicted in Table 1.

2.2. Structural analysis

Figure 1. and Figure 2. shows the XRD patterns of PbS samples prepared on glass substrate and on PET substrate respectively. The XRD patterns illustrate that the films coated on PET substrate was clearly crystallised without any unwanted peaks due to hydroxide or other impurities (below 20^0). Whereas the film coated on glass substrates shows several peaks indicating the presence of hydroxide contents. So the film formed on PET substrate was chosen for further structural, morphological and optical studies.

XRD pattern of the film on PET substrate show peak corresponds to (111) crystal plane of cubic structure of PbS. This was confirmed by using

Sample	Thickness (nm)	Microstrain	Band gap (eV)	Grain size (nm)
CPbS-PET	1100	0.001227	1.10	7.27

Table 1: Thickness, microstrain, band gap and grain size of PbS-PET film

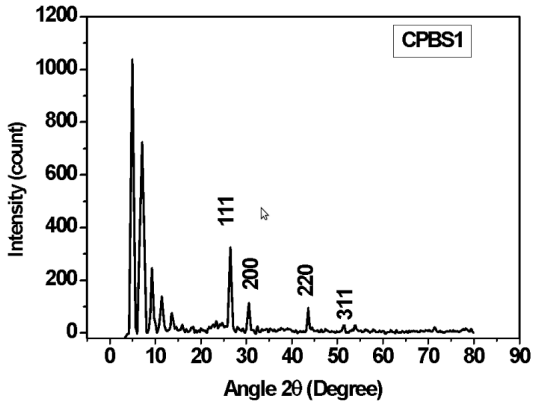

Figure 1: XRD patterns of as grown PbS thin films on glass substrate

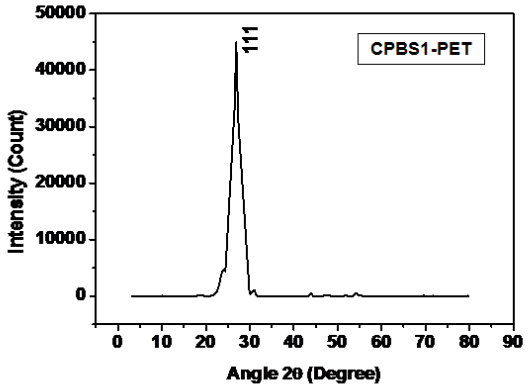

Figure 2: XRD patterns of as grown PbS thin films on glass substrate

Sample	**Lattice constant a(A^0)**	**2θ(^{o})**	**d(A^o)**
Ideal	5.9392	26.034^o	3.4199
CPBS1-PET	5.7288	26.935^o	3.3075

Table 2: PbS-PET films with crystalline size, lattice constant, 2θ and d value

JCPDS data. The very high intensity of the diffraction pattern indicates the high crystallinity of the as grown sample.

Peak exhibited by the sample is very broad. The occurrence of broad diffraction peak signifies that the thin films fabricated on PET substrates are made up of nano-size crystals. No characteristic peak of other phases or impurities was observed in the XRD pattern.

The particle size of the CPBS1-PET crystallites were determined by Deber-Sherrer formula and are depicted in Table 1. The particle size obtained is below the Bohr excitonic radius of PbS (18 nm). This is an indication of quantum confinement.

Microstrain depends largely on process parameters. The strain formed may be correlated the lattice misfit, lattice expansion or contraction, which in turn depends up on the deposition conditions. Microstrain in the nanocrystals of PbS film was computed by using the tangent formula and is depicted in the Table 1. Strain developed in the CPBS1-PET film is minimum and negligible. Preferential orientation of the films was resulted due to this negligible strain. The use of PET-substrate for film synthesis might have reduced the strain developed.

2.3. Morphological analysis

Figure 3 shows SEM images of the CPBS1-PET film. As shown in Figure 3, lead sulphide crystallites of various sizes uniformly carpet the substrate surface. In addition, tiny spherical structures are dispersed on the film's surface, which seems to be

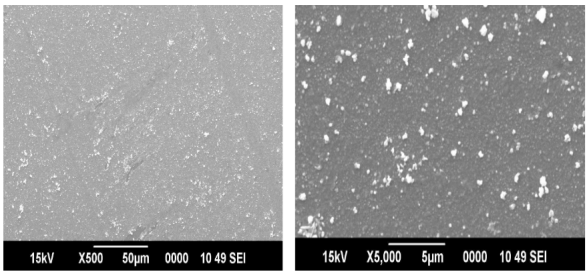

Figure 3: SEM images of the CPBS1-PET film

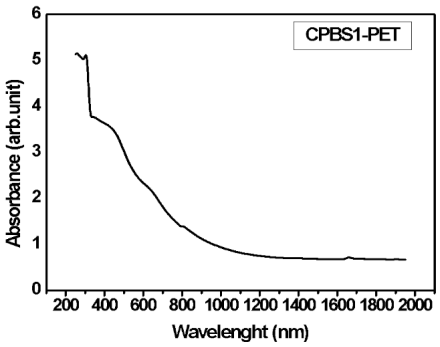

Figure 4: Absorption spectra of as grown PbS film

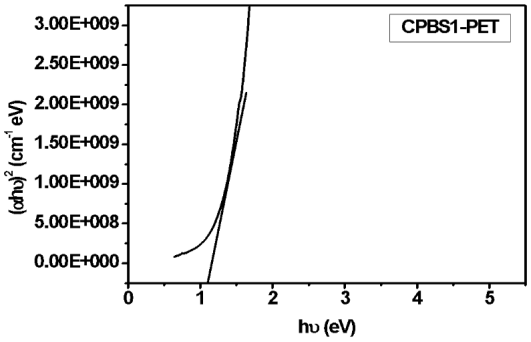

Figure 5: The $(\alpha hv)^2$ Vs (hv) plot of as grown PbS film

Sample	Band gap
CPbS-PET	1.10 eV

Table 3: Optical band gap of the PbS thin film

made up of by cluster of nano particles. Their preferential orientation along (111) direction can be attributed to the coalescence of crystallites on the thin film surface. It is illustrated that the film developed on the PET substrate was well crystallized, with good surface homogeneity and roughness.

2.4. Optical studies

The absorption spectra of as- synthesized CPBS1-PET film is shown in Figure 4. In the UV region of the spectrum, the film is highly absorbing and the absorbance is gradually decreasing with increase in wavelength in the visible region.

The absorption spectrum illustrates that CPBS1-PET film has high absorbance in the visible region, indicating applicability as an absorbing material. In the Near IR region the value of absorbance is low. The optical band gap of the film was determined by plotting $(\alpha hv)^2$ vs (hv) as shown in Figure 5. Accordingly, the energy band gap is obtained by extrapolating the straight line portion of the curve to the hv - axis. The value of optical band gap obtained is given in Table 3. The band gap of the CPBS1-PET film obtained is 1.10 eV and clearly indicates evidence for a blue shift from the optical band gap 0.41 eV of the bulk PbS crystals. Similar results were reported in the literature.

Figure 6 and 7 respectively shows the optical reflectance and transmittance spectra of as grown CPBS1-PET film. Both the spectra show a wavy nature. Transmission is increasing with increasing wavelength towards the near infrared region. In the lower wavelength region, there is no transmission since all the light is absorbed by the film.

The Figure 8 and Figure 9 respectively shows the variation of extinction coefficient and refractive in-

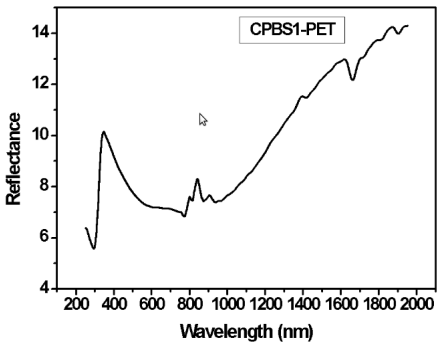

Figure 6: Reflectance spectra of as-grown PbS thin film

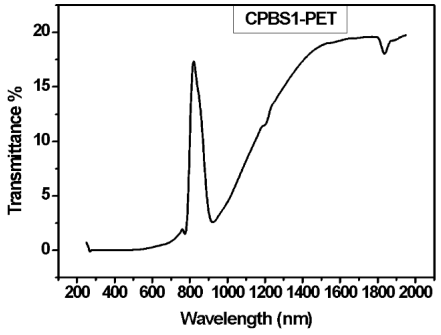

Figure 7: The transmittance spectra of as grown PbS film

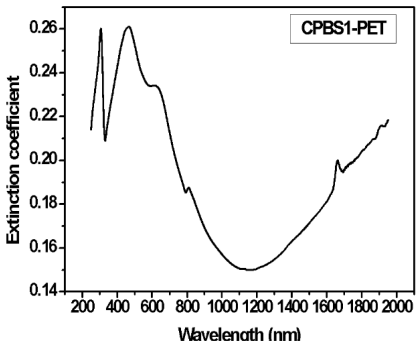

Figure 8: The variation of extinction coefficient with wavelength

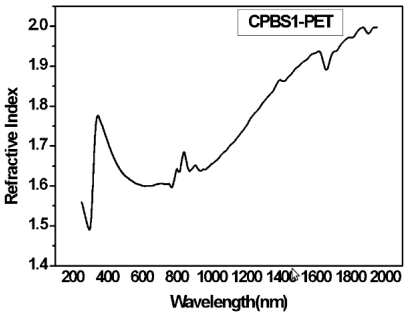

Figure 9: The variation of Refractive index with wavelength

dex with wavelength. Extinction coefficient is high in the UV-Visible region and reduces to a low value in the NIR region. It further increases as the wavelength increases. Refractive index also varies in a similar manner.

3. Conclusion

PbS thin films were deposited on properly cleaned glass and polyethylene terepthalate (PET) substrates by chemical bath deposition (CBD) technique at room temperature. The films were prepared by using Lead Acetate Trihydrate $Pb(CH_3COO)_2.3H_2O$, Thioacetamide C_2H_5NS, Triethanolamine (TEA) $C_6H_{15}NO_3$ and distilled water. The structure was determined by X-ray diffraction studies. The surface morphology was determined by scanning electron microscope measurements. The optical properties were carried

out from spectroscopy measurements of absorption-transmission-reflection.

The XRD studies illustrated that the films coated on PET substrate was well crystallised without any peaks due to hydroxide or other impurities. The film coated on glass substrates shows several peaks indicating the presence hydroxide contents. The films were very adherent to the PET substrate and well crystallized according to the centered cubic structure with the preferential orientation (111). The crystallite size 7nm of the film is below the Bohr excitonic radius of PbS and showed quantum confinement.

Microstrain developed in the PbS film deposited on the PET substrate is minimum and negligible. SEM images illustrated that lead sulphide crystallites of various sizes uniformly carpet the substrate surface. The optical band gap of the film obtained was 1.10 eV and exhibited a blue shift compared to that of the bulk PbS crystal. In the lower wavelength region, there is no transmission since all the light is absorbed by the film. Transmission is increasing with increasing wavelength towards the near infrared region.

This material can be used for solar absorption in solar cell applications. In addition, PbS thin film has been utilized as decorative and solar control coatings.

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Studies on Isolation and Characterisation of Humic Acid From Various Sources

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Abstract

In the present study humic acid (HA) were extracted from various sources like soil compost and sediment. The isolated HAs were purified further and fluorescence spectra were taken. The fluorescence behavior of compost derived HA suggested that it has the lowest molecular size and aromatic condensation content. Soil HA has intermediate behavior compared to the other two HAs. This may be due to the short period of humification that takes place in the composting process.

1. Introduction

HA, the most resistant fraction to microbial degradation of the organic matter in soil, are complex polymeric organic acids with a wide range of molecular weight. They are heterogeneous mixture of a variety of organic compounds, consisting of aromatic, aliphatic phenolic and qunolic fuctional group with varying molecular size and properties. In addition to size chemical composition, structure and functional group of HA may vary greatly depending on the origin and age of the material. They affect a variety of chemical, physical and biological reaction.

Organic matter, compost, humus, humates, humicacid, fulvic acid are all related to and are parts of decaying plant materials. These organic matters are food for soil life and store house for minerals, energy and water. They also serve as medium on which certain organism can grow.

For many years the most commonly used humic product was a black liquid extract called HA, obtained by mining a strong base liquid such as sodium hydroxide or KOH, with a dry humic material. Usually a 6 or 12% solution was most commonly mixed with liquid N sources for use in transplant solution or mixed with herbicides. Fulvic acid another extraction from dry humates, is truly an acid. It is an acid extraction and has pH near 3.

Chemically HA contains more C and less H. It also contains about 4% N. But does expect this N to be of any consequence as far as plant growth is concerned. Because HA is one of the end products of the biological decay of organic matter, it has a great resistance to biological decay of organic matter, it has high resistance to further decomposition. Estimates of its microbial decay rate are often in the range of 0.3% per year under ideal laboratory conditions(Aiken etal 1985).

Two properties of HA are cation exchange capacity to form chelates with metallic micronutrients, Fe, Cu, Zn and Mn. The cation exchange capacity of commercially produced HA are in the range of 500 to 600 milli equivalents per grams. The chelating action of HA is sometimes used to produce chelated iron products. Without the addition of nutrients such as Iron, the claim is often made that HA has the ability to. Solubilize micronutrients already in the soil.

Naturally occurring and commercial humic sub-

stances have been shown also suitable for soil bioremediation, biodegradable and non toxic for soil microbial biomass. These properties makes the HA highly desirable also as candidates for technological applications in many fields of chemistry, agriculture soil science. Such perspective appears particularly intriguing now days in relation to the current issue of fossils versus renewable sources of chemical compound (Velesco etal 2004).

According to elemental analysis and molecular weight measurements, NMR and IR spectroscopic data, the HA molecular structure containing 26 phenyl propyl repeating unit ,each on bonded to short or long hydrocarbon chains of come up to an average of total 30 carbon atoms per repeating unit. Several functional polar groups have been identified in this polymeric molecule i.e.; NH_2, CONH, COOH, XOH, XOX, X=alkyl or phenyl. The whole structure appear well consistent with the good performance of the HA matter as surfactant. A mostly intriguing aspect is that such structure is available by natural process.This fact make the above HA material very competitive for the low cost, in addition to the good properties. Since the structure of humic like substances vary greatly as a function of compost ingredients and composting time ,composed wastes appear a very rich sources of chemical or ionic materials for applications in many fields of chemistry, agriculture etc. HAs are one of the most active fraction of OM (organic matter) they improve the absorption of nutrients by plant and soil microorganism (Chen J etal 2003).

2. Materials and Methods

Compost, soil and sediment dried HAs were extracted with a slightly modified procedure recommended and used by the IHSS (Inter National Humic Substance Society) to isolate standards of HA.

10g sample is treated with 0.1 N HCl .The suspention was shaken for 4h .The residue was neutralized to pH 7 with NaOH . Added 100 ml 0.1 NaOH. Shaking 8h .collect the supernatant and is acidified using HCl and allowed to stand for 24 hrs. The HA fraction was redissolved by adding 0.1N KOH. And remove suspended solids .process repeated and HA precipitate is suspended in a mixture of 0.1N HCl 0.3N HF solution in a plastic container and shaken overnight at room temperature. The HA was washed with distilled water and dried.

Humic acids	**E4**	**E6**	**E4/E6**
Compost	0.678	0.127	5.34
Soil	0.667	0.158	4.22
Sediment	0.487	0.116	4.20

Table 1: E4/E6 value of HA studied

10Mg of HA was dissolved in 10 ml 0.036N NaOH .For fluorescence analysis stock solution was diluted to 100 times. HA solutions were analysed after 10 days of preparation because it was found that after this time solution stability is reached. The sample were further analysed by fluorescence spectra, using shinadzu photoluminescence spectrofluoro photo meter RF5301PC.Emission spectra were recorded over a scan range of 400-700 nm by measuring fluorescence intensity at a fixed excitation wave length of 355 nm.

3. Result and Discussion

Optical densities of HA solutions were measured at 465 nm (E4) and 665 nm (E6) (Table 1).

The magnitude of E4/E6 is related to the degree of condensation of the aromatic C network, with a low ratio indicative of a relatively high degree of condensation of aromatic humic constituents. Conversly, a high E4/E6 ratio reflects a low degree of aromatic condensation and suggests the presence of relatively large portions of aliphatic structures. Compost derived HA, has a high E4/E6 ratio reflects a low degree of aromatic condensation and suggests the presence of relatively large portions of aliphatic structures (Chen etal 1977and Kononova 1966).

Low degrees of aromatic C network and low average molecular weight, probably due to a short period of humification that takes place in the composting process. The ratio is an indication as humification as the degree of condensation of aromatic rings in humic substances. The lower value the greater is the humification.

Figure 1. Compost HA showed only one broad shoulder in the region 400 to 600nm unlike those observed in the corresponding spectra for soil HA. Composed HA has a higher intensity than the soil HA and sediment. The maximum is observed at 450nm. The fluorescence behavior of compost HAs

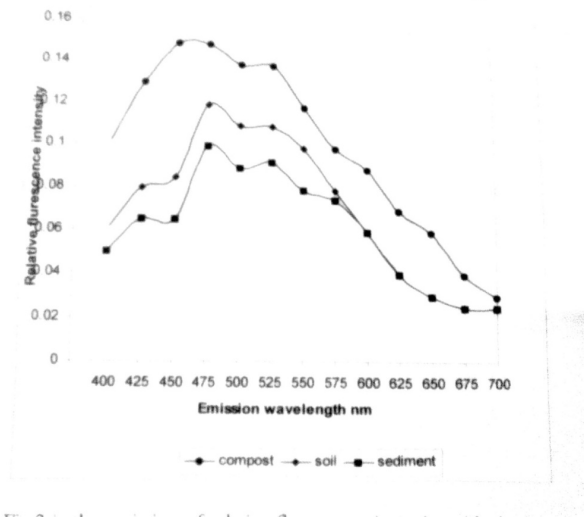

Figure 1: Comparison of fluorescent spectra of HA

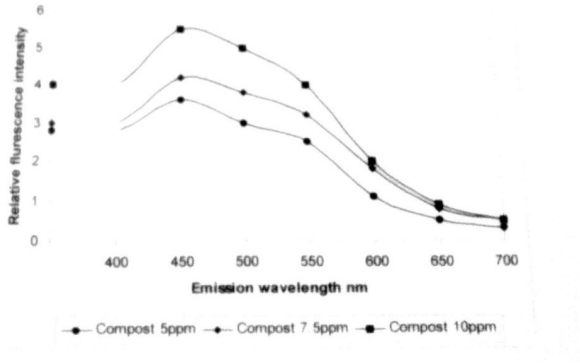

Figure 2: Effect of HA (compost) concentration on fluorescent spectra

studied in this work indicates that this CHA have the lowest molecular size and aromatic condensation content because of the short period of humification that takes place in the composting process. These compounds are HAs in neoformation. In this way, CHA can be more easily biodegradable than soil HAs, but when composts are used as soil organic amendments, their HAs in neoformation can be incorporated in the soil HAs structure and contribute to chemical and physical fertility and they increase the soil organic matter content (Mikki et al. 1997).

Figure 2. is the variation of relative florescence intensity with the HA concentration extracted from compost. It showed an increase of relative florescence intensity with an increase of HA concentration on 5 to 10 ppm.

4. Conclusion

The fluorescence behavior as well as optical density of compost has studied in this work indicates that this CHA have the lowest molecular size and aromatic condensation content because of the short period of humification that takes place in the composting process. These compounds are HAS in neoformation. In this way, CHA can be more easily biodegradable than soil has, but when composts are used as a soil organic amendment, their HAs neoformation can be incorporated in the soil HAs structure and contribute to chemical and physical fertility and they increase the soil organic matter content.

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In-Migrant Construction Workers In Kasaragod District Of Kerala: Livelihood Issues And Adaptation Strategies

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Abstract

The present enquiry is an attempt to explore the livelihood issues and adaptation problems of in-migrant construction workers in the context of Kasaragod by exploring the basic profile of in-migrant workers in Kasaragod district and assessing the livelihood issues encountered by in-migrant workers in the study area. The study has found that the in-migrant construction workers in the district face many issues which require serious policy interventions.

1. The Context

The movement of people in search of better economic conditions and a more secure environment is as old as human history. Such movements not only affect the lives of the migrants profoundly, but also lead to significant economic and social transformation in the regions of origin and destination of the migrants (Narayana, 2013). Migration of people from one place to another is a complex phenomenon. It has multiple dimensions and differs according to class and social groups in developing countries. The process of migration is changing very fast, particularly in globalization era which is characterized by structural changes and consequent alterations in the economy as a whole and in rural economy in particular. Hence, the nature, pattern and magnitude of migration have been evolving overtime. Migrant workers constitute a major category of migrants in general. The International Convention on the Protection of the Rights of All Migrant Workers and Members of Their Families has defined a migrant worker as 'a person who is to be engaged, is engaged or has been engaged in a remunerated activity in a State of which he or she is not a national'. The United Nations Multilingual Demographic dictionary defines migration as a form of geographical mobility between one geographical unit and another, generally involving a change of residence from the place of origin to place of destination or place of arrival. Therefore, migration involves the 'detachment from the organization of activities at one place and the movement of the total round of activities to another' (Goldscheider, 1971).

People started moving across borders in search of better opportunities and jobs for several decades. However, labour migration was speeded up and popularized during the era of globalization. Accurate data on migration is difficult to obtain because many migrant workers lack official status. According to an estimation of UN Department of Economics and Social Affairs, there were around 200 million international migrant workers and their families in 2010. Due to lack of employment opportunities in developing countries and increased demand for low wage workers in developed countries, people are pursuing work in other countries in order to support themselves and their families back home.

According International Labour Organization (ILO) estimates, out of the 175 million migrants worldwide, 120 million are migrant workers and their families. Today, ILO estimates, there are roughly 20 million migrant workers, immigrants and members of their families across Africa, 18 million in North America, 12 million in Central and South America, 7 million in South and East Asia, 9 million in the Middle East and 30 million across all of Europe. Western Europe alone accounts for approximately 9 million economically active foreigners along with 13 million dependents. Migration has become a key policy issue for many developing countries, depending on the state of the national economy and the employment situation. In India, people migrate from rural to urban areas mainly due to poverty, high population pressure on land, absence of infrastructure facilities etc. There are also many other factors like natural disasters and local conflicts that give extra push to migration. On the contrary, there are several pull factors which attract people from rural areas to urban centers. The most important pull factors are better opportunities, higher wage rate and availability of regular work in urban areas. Availability of better education and health facilities and sources of entertainment are also considered as major pull factors. The remittances play vital role in the economic development of migrant families to a great extent.

2. In-migration to Kerala: Possible Inferences

Kerala is well known for the outmigration of labour to the Gulf countries. But in the recent years Kerala is witnessing large inflow of migrant labour from different parts of the country. It is interesting to note that while many *Malayalees* migrate to the gulf countries (Middle East) both for skilled and unskilled work, many of the unskilled labour from other part of the country consider Kerala as their Gulf. Moreover recent boom in hospitality and real sector has crafted a construction sector boom in Kerala. The out-migrations of labour to gulf countries from Kerala and recent construction sector boom have led to chronic shortage of labour in the construction sector. The chronic shortage of labour felt in the construction sector in our state and the resultant higher wage rate received the attention of workers in other states and they began to move to Kerala in search of work. With signs of rapid growth of state's economy and the increase in activities particularly in the construction sectors, the in-migration is expected to grow faster in the coming years. Within the State, Kasaragod a Northern District which has been recently emerged as a significant pocked of in- migrant construction workers. Moreover Kasaragod district is culturally diverse, economically backward and policy neglected district of Kerala. As large number of people from Kasaragod District has gone out to other countries for better jobs and majority of our educated youth go for preferring white color jobs, there is a huge gap in the demand and supply of labourers in the job market. This widening gap is very often filled by the migrant workers from other Indian states. While a relatively small section of migrants from other states are professionals and skilled workers, large majority are unskilled workers engaged in construction works. The construction industry is the first point of entry for the migrants to the study area. Thus the shortage of local labour, higher wages for unskilled labour in the state and better opportunities for employment led to the massive influx of migrant labourers. In the literature surveyed so far we find that there is no work on livelihood issues and adaptation problems of in-migrant workers in Kasaragod district of Kerala. So the present study is an attempt to explore the livelihood issues and adaptation problems of In-migrant construction workers in the study area In the light of the stated problem, the present enquiry attempts to explore the basic profile of in-migrant workers in Kasaragod district, assess the livelihood issues encountered by in-migrant workers in the study area, examine the adaptation problems of in-migrant workers to cop up with local condition and to examine the health status of in-migrant construction workers in the study area.

3. Methodology

3.1. Area of Study

The area of study is confined to Kasaragod District of Kerala, which has been recently emerged as a significant pocket of in-migrant construction workers. The study assumes relevance in the context that the labour market of the district can never be delinked from the in-migrant construction workers.

3.2. Sampling Design

As the study is more primary data intensive, more care has been taken while dealing with first-hand information. For avoiding the later filtering processes a primary pilot survey has been organized for selected 20 workers. The study runs through a total sample of 60 in-migrant construction workers, not holding a ration card of the state. Construction sites and workers have been identified on **a snow ball sampling basis**[1] , for which the necessary assistance of local people has been sought at each level. For not surrendering any degree of authenticity workers have been met at two points - at the work site and at their residence. The work-site interview has been scheduled and executed at the convenience of the employer, contractor and the workers. The household survey has been conducted between 6 to 8 pm.

3.3. Sources of Data

The present enquiry relies on both primary and secondary data sources. The primary data on in-migrant construction workers were collected from Kasaragod district. The secondary data were collected from Census 2001 and 2011, NSSO Report, Economic Review of Kerala and the publication of Labour Rehabilitation Department of Kerala etc. A detailed questionnaire is drafted for the purpose of data collection. The questionnaire may contain questions related to socio-economic conditions of in-migrant construction labourers, details of working conditions, savings and consumption habits, job security and job accessibility, health issues, details about accommodation facilities, major problems faced by migrants in Kerala, policy suggestions etc.

3.4. Tools of Analysis

The analysis relies on simple statistical tools like Average, Correlation, Ratio, Percentages and Simple Growth Rate. For generating frequencies and cross tabulations the SPSS package is made use of. The Ms-Excel package is used for generating suitable and meaningful pictorial representations like Bar Diagrams, Pie Diagrams, Line Chart and the so forth.

[1]The snow ball sampling procedure of the identification of the sites and workers starts with identifying either a contractor or a worker and catching hold of the other one. In the later stages sample develops by gathering information regarding other contractors and workers from the already known.

4. Major findings of the Study

It is found that maximum number of migrants (31.11 percent) came from West Bengal followed by Bihar and Assam (17.77 and 12.22 percent respectively). More than half of the migrant construction workers have only primary school education where as one third of them had no formal schooling. Major percentage of the migrants (75.56) belonged to Hindu religion. It has been found that 24.44 percent of workers belong to the age group of 25 and 30. It can be seen that 65 percent of workers belong to the age group of 20 and 35. Around 4 percent are above 40 years of age. Surprisingly 6.66 percent workers are below 18 years of age and another 7.78 percent between 18 and 20 years. Thus around 14 percent workers come under 20 years of age. Family size migrant workers seem to be large with majority having 5 members at home and almost one fifth are having more than five members at home. It is noticed that more than 84 percent of the respondents are unable to handle Malayalam while only the remaining few can somehow able to understand the language used in Kerala.

Majority of migrant workers are engaged in unskilled works like doing construction requirements of various sites. It is noticed that the more than one - fourth of the migrant construction workers do not possess land and house in own village. Regarding the wages received per day, majority of skilled workers receive remuneration between Rs 400 and 500 while majority of unskilled workers receive wages between Rs 200 and 300. They are also seeking loan/advance facilities from employer but less number of migrants gets loans and advances from their employers. For remitting money to home the most important method accepted is banking facilities. The push and pull factors of migration to the study area are examined. The consolidated data of the respondents reveal that 41.12 percent of the migrants came to the district due to the fact that the existing wages in their own villages were very low. Another 28.89 per cent came here due to lack of job opportunities in their village. To 26.67 percent, poverty was the major push factor. The high wage rates and more employment opportunities were the key factors that pulled them to

Kerala. Maximum number of workers has been doing construction work in the study area between 2 and 5 years. Very less number has completed more than 5 years in the study area. The average number of hours of work per day is found to be nine hours per day. As high as 82 percent of the workers reported that they have to work for more than 8 hours on a working day. It could be noticed that 82 percent of the migrants in Kasaragod are doing overtime work. The wage received by the migrant workers differs in accordance with the nature of work they do. The skilled and unskilled workers are paid differently. Among the unskilled category, there are wage differences in various work sites and different geographical areas. The mode of payment varies among different employers. The most common mode of payment is found to be weekly payments of wages.

Most of the migrant workers expressed satisfaction in getting promised pay in time. However there is an exception that a few of them are not given payments in time. It can be noticed that lions share of migrants do not receive festival allowance. Only around 23 percent workers get allowances while they stay in Kerala during festival seasons. 89 per cent of migrants are engaged in construction works without any training. Only 11 percent of the workers get training especially the group engaged in skilled works. Employers are not alike in providing sick leaves because once they get a chance to leave the work sites; it may be difficult to bring them back to the same job. The tendency of leaving for better chance is also common among migrant workers. Data reveals that 58.5 percent of the migrants get sick and other kinds of leaves both to take rest as well as going home for treatment. A major issue faced by the migrant workers is related to harassment in the work sites. It has been seen that one fourth of the migrant workers have experienced regular harassment in the work sites by the supervisors. Migrant workers in the study are not having any trade union membership. It has been observed that 75 per cent of the migrants are staying at the work site itself. It can be found that 36.66 percent workers stay in rooms with 5 persons. Another 13.33 percent stay in rooms with 6 persons and 10 percent migrants stay with 7 persons. It is also noticed that 12 per cent of workers live in rooms where more than 7 persons reside.

The most pathetic situation related to overcrowded stay is the toilet facilities. Most probably the toilet facilities are lacking in connection with their rooms. Instead they depend on common bathroom/toilets shared by 10-15 persons. 64 percent of the workers get food by the employers. It is found that around 87 percent workers go for purchasing from private shops in nearby places. Most of the migrant workers to be precise, 88 percent are not having any chronic diseases. It is revealed that 68 percent workers had come across some kind of accidents and work related illness. This includes accidents in connection with construction works, chronic allergic problems, severe and continuous headaches while in the sun light for long hours etc. Around 81.5 per cent workers depend on government hospitals because of the low cost or free treatment provided. Three percent workers have used to consult private hospitals. It is noticed that 12 percent construction workers used to consume medicine directly from the medical shops without consultation of doctors. 69 per cent of the construction workers do not have any health insurance. Only 31 percent are having health insurance/ group insurance in the study area. 58 per cent of the migrant workers have very less communication with local people. 23 per cent of the workers do not have any communication with local people. The workers are of the opinion that the local people of the study area very often hesitate to communicate with them even if they take initiative. 81.4 percent of the migrant construction workers in the study area do not have knowledge in Malayalam. Language barriers and inadequate communication with the local people are serious adaptation problems of in-migrant construction workers in our district.

5. Recommendations

From the results emerging from the Study, there is clearly a need to address the following issues urgently. The recommendations under the areas such as migrant registration, improvement of housing and living conditions, social security and health issues, employment issues, support through help-line etc. There should be proper registration system for the migrants. Registration card with unique number should be issued to migrant workers and it should be computerized systematically to make future references. Health cards should be provided to migrant workers and they should be included in

the health insurance cover. There should be provision for the migrant workers to join in the trade unions. They can thereby realize their rights and needs and can get away from the harassments and exploitation of their employers. The migrant workers should also be brought under the ambit of the Construction Workers Welfare Fund Board and enable them to avail of the social security provisions. The benefits of all Government schemes will be conditional on registration. The number of working hours should be strictly restricted to 8 hours per day. Child labour in the construction sector should be strictly prevented. There should be government mechanism to monitor assurance of minimum wages, festival allowances etc. Facilities for job training should be arranged for migrant workers It is important that the State Government initiate steps to provide affordable group housing to the migrants in the state. Along with housing facilities, associated services such as water, electricity, sanitation, washrooms, ensuring a green environment etc. should be provided to the migrant workers. It should be compulsorily made sure that the migrants have adequate toilet facilities in crowded centers.

Private enterprises with public support may be encouraged to build and provide hygienic accommodation with electricity, running water, sanitation facilities, sufficient toilets, washrooms etc. on reasonable rent to migrants especially in the major cities and towns or nearby areas of major concentration of migrant workers. The State Government should have a Social Security schemes for the migrant workers in the construction sector in the State. The Social Security Schemes should include provision for payment of lump sum to take care of in cases of accidents and serious illness, death, loss of limbs etc. A help-line staffed by people speaking different languages and with separate numbers for different languages should be a top priority. Support mechanism for hearing migrant complaints should be set up at local government level. Awareness programmes regarding the rights and obligations of migrant workers should be organized in their languages such as Bengali, Hindi etc. and telecasted through radio, TV and displayed in public places including railway stations. Coordination between host state, states of origin and the central government may be necessary in other aspects such as extending the benefits of the public distribution system, sharing of information on migrant workers who are sent back to home state when infected with contagious diseases like Malaria, incidence of such diseases in the states of origin etc. There is also a need for considering migration as one of the criteria in central allocation to states. Some of the programmes for creating awareness about the rights in destination state can be organized in the states of origin. It is also possible that the states of origin can contribute to the welfare funds for migrant workers implemented in the destination state to make it more attractive. There is also a need to strengthen the public facilities taking into account the increasing need of the migrant population. Government should take initiatives to integrate the domestic migrant workers with the local population by developing common platform of interactions through cultural exchange Programme and specific schemes dedicated to the socio-educational and cultural development of domestic migrant population.

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Financing Of Revenue-Expenditure Gap: The Experience Of Nileswaram Municipality In North Malabar Of Kerala

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Abstract

The present enquiry aims to look at the fiscal health of Nileswaram Municipality, an emerging municipality in Kasaragod district of Kerala by estimating the revenue expenditure gap at various levels and assessing the mechanism of financing the gap. Even though the overall revenue-expenditure gap was found to be in surplus across all the years reported, the estimated gaps at the disaggregate levels shows huge deficit; representing unhealthy fiscal conditions of the selected municipality. The current revenue-current expenditure gap was highly unfavourable and continuously growing. Apart from that, it has been found that the municipality is increasingly depending upon the General Purpose Fund for matching the current revenue-current expenditure gap. This leads us to the conclusion that the municipality will be under severe fiscal stress if the General Purpose Fund does not increase much to meet the increasing expenditure requirements of the municipality. It has also been observed that the growth rate of General Purpose Fund is well under the growth rate of Current Revenue-Current Expenditure Gap, which needs to be looked at seriously by the municipality in the coming years.

1. The Background and the Larger Context

Financially sound Urban Local Bodies is a prerequisite for urban development with well managed cities and improved living standards of urban masses, especially in a rapidly urbanising and developing economy like India. The interventions by the Urban Local Bodies significantly influence the well being of urban masses; by providing them with necessary socio-economic infrastructures and civic amenities (Mohanty et al, 2007). Urban Local Bodies are forced by the increased public demand for various urban civic provisions to actively indulge in the provision of such services. Apart from that, in the era of globalisation, urban areas are demanded by the circumstances to remain as a vibrant and dynamic centre of competitiveness. However, inadequate revenue sources pull the Urban Local Bodies under severe fiscal stress, which in turn constraints the capacity of cities as engines of growth (Mohan, 1996; Mohanty et al, 2007).

The discussion of revenue inadequacy of local bodies goes back to the 74^{th} constitutional amendment wherein the functional domain of the Urban Local Bodies has been clearly demarcated in the Twelfth Schedule with no mention about the revenue sources for meeting the listed responsibilities; and the assignment of finances to the local bodies has been left to sole discretion of respective state and the central government (Mohanty et al, 2007). This has generally resulted in a trend of local finance in general and the urban local finance in particular varying across states with wide mismatch between the functions assigned to them and the resources available for discharging their responsibilities (Mohanty et al, 2007). Thus Urban Local Bodies are forced to depend upon the state governments

for assigning revenue sources and provision of inter governmental transfers from central government for discharging their responsibilities (Soumen, 2009). No need to emphasise that the fiscal health of the Urban Local Bodies is necessary for their smooth functioning and for financing various developmental activities in the urban areas. The fact that existing resources in the urban areas are also not properly tapped and utilised adds fuel to the fire and the urban local bodies have to face severe fiscal stress, failing to finance urban infrastructure and other basic amenities. The story at the national level and also at the state level may not be sufficient enough to go for a concrete argument regarding the fiscal health of Urban Local Bodies; rather micro level analysis can provide a better picture. The present enquiry in this context attempts to assess the revenue-expenditure gap of Urban Local Bodies with special reference to Nileswaram Municipality in the North Malabar of Kerala, which is an emerging municipality in Kasaragod district of North Malabar. The paper is outlined in seven sections including the context. The second section deals with the measurement issues in the concept of revenue expenditure gap. The level and determinants of revenue expenditure gap at the aggregate and disaggregated levels are discussed in the third and fourth sections respectively. The fifth section deals with the financing of revenue expenditure gap. Sixth section deals with the components of the revenue-expenditure gap and the final section concludes the discussion.

2. Revenue-Expenditure Gap: The Concept and Measurement Issues

The financial health of the Urban Local Bodies can be better assessed by comparing expenditure heads with the revenue sources. Hence, an appropriate measure of revenue-expenditure gap would serve the purpose. The gap can be measured at two broad levels: aggregate Level and the disaggregated level. At the aggregate level gap is measured as the simple difference between Total Revenue (TR) and Total Expenditure (TE). At the disaggregated level the gap can be measured either as (i) the difference between **Own Revenue** (OR) and **Current Expenditure** (CE) or as (iii) the difference between **Current Revenue** (CR) and **Current Expenditure** (CE). The gap has been normalised to the base by converting the same as a percentage of revenue. The aggregate revenue expenditure gap takes the following form.

$$Gap(Aggregate) = [TR - TE]/TR \times 100 \quad (1)$$

The revenue expenditure gap at the disaggregated level captures the fiscal position better as it does not include the influence of conditional revenue in the revenue side of the municipality. Hence at this level the gap measures the difference between revenue generated by the municipality and the expenditure to be incurred from such sources. At the disaggregated level the revenue expenditure is estimated further at two levels.

$$Gap(Disaggregate1) = [OR - CE]/OR \times 100 \quad (2)$$

Own revenue includes own tax and own non-tax revenue. Income from other sources like interest earned is not included in the own revenue. The present measure captures the gap between revenue mobilised from standard own revenue sources (like tax and non-tax sources like fees and user charges, rental income from municipal properties, sales and hire charges and so on) and expenditure to be met from own revenue sources. The second disaggregated measure is an adjusted measure of the first one. It considers current revenue (CR) instead of own revenue, wherein current revenue is nothing but own revenue plus income from other sources like interest earned. This could take the following form;

$$Gap(Disaggregate2) = [CR - CE]/CR \times 100 \quad (3)$$

3. Revenue-Expenditure Gap at the Aggregate Level: Levels and Determinants

Gap in overall revenue and expenditure can give a crude picture of municipality's fiscal position; as most of their revenue is derived from revenue grants, which are mostly conditional in character; except revenue grants in General Purpose Fund. The overall surplus may be the surplus in revenue grants which cannot be used for any other purpose than for which they have been sanctioned. Nileswaram municipality has been able to raise revenue of around 7.3 Crores and only approximately

Table 1: Revenue-Expenditure Gap at the Aggregate Level

Year	Reveneue	Expenditure	Gap	Gap (% of Revenue)
2011-12	73303935	50142260	23161675	31.6
2012-13	74739360	71162281	3577079	4.8
2013-14	89573722	88822004	751718	0.8
2014-15	103489718	99556757	3932961	3.8

Source: AFS of Nileswaram Municipality from 2011-12 to 2014-15

Table 2: Growth in Revenue, Expenditure and Gap

Year	Growth Rate of Revenue	Growth Rate of Expenditure	Growth Rate of Gap
2011-12	-	-	-
2012-13	2.0	41.9	-84.6
2013-14	19.8	24.8	-79.0
2014-15	15.5	12.1	423.2

Source: AFS of Nileswaram Municipality from 2011-12 to 2014-15

5 crores have been expended during 2011-12, leaving a huge surplus of around 2.3 crores (refer to table 1). The absolute gap has come down in the two subsequent periods to 35 Lakhs and then to 7 Lakhs during 2012-13 and 2013-14 respectively. In all the periods reported the revenue expenditure gap has never been deficit. During 2011-12 the surplus was around 31.6 per cent of the total revenue and the proportion has come down to 4.8 by 2012-13, then to 0.8 during 2013-14 and subsequently to 3.8 during 2014-15. Hence, a general trend of decline in surplus could be seen over the years. Even though there is surplus at the overall level, it could be inferred here that the fiscal position during the initial years was much better and the situation started worsening from 2012-13 onwards. The gap (surplus) has come down from 2011-12 to 2013-14 due to the faster increase in expenditure than revenue. While revenue has grown at a marginal rate of 2 per cent, expenditure has grown at 41.9 per cent from 2011-12 to 2012-13 as it could be inferred from table 2. From 2012-13 to 2013-14 the growth rate of revenue was to the tune of 19.8 per cent and that of expenditure was 24.8 per cent. But, from 2013-14 to 2014-15 the surplus has increased because expenditure growth rate has fallen at a faster rate than the growth rate of revenue.

4. Revenue-Expenditure Gap at Disaggregated Level: Levels and Determinants

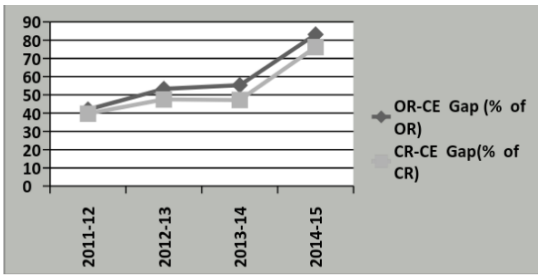
The logic of estimating the first gap is that the local bodies are supposed to meet their current expenses from own revenue sources. The second measure adds other income sources which are also not conditional to the own revenue to arrive at current revenue. But, nevertheless since other revenue sources are not that significant as far as municipalities are concerned, there won't be much difference between both the measures, as it could be visualised from the table 3. The own revenue of Nileswaram municipality was found to be well short of expenditure to be met from own revenue (current expenditure) in all the years reported. The deficit was even approximately one Crore for all the years reported. The gap was found to be continuously growing throughout all the years reported; and the trend does not show any difference between the two measures since other income sources are less significant as far the selected municipality is concerned (refer to figure 1). The inference, hence, will be the same whichever measure of revenue expenditure gap (Own Revenue-Current Expenditure Gap or Current Revenue-Current Expenditure Gap) is considered. The Own Revenue-Current Expenditure Gap was to the tune of 42 per cent of own revenue during 2011-12 as it would be inferred from table 3. When other income is added to the own revenue the gap slightly falls down to 39.7 per cent (CR-CE gap). The gap has gone up to 53 per cent during 2012-13 and to 55 per cent during 2013-14. The gap is found to be drastically widened to 83 per cent of own revenue during 2014-15. The gap has slightly been low when other income is added along with own revenue; as could be seen from the Current Revenue-Current expenditure gap figures. While Own Revenue-Current Expenditure Gap was to the tune of 83.1 per cent, Current Revenue-

Table 3: Revenue-Expenditure Gap (OR-CE & CR-CE Gap)

Year	Own Revenue	Current Revenue	Current Exp.	OR-CE Gap	CR-CE Gap	OR-CE Gap (% of OR)	OR-CE Gap (% of CR)
2011-12	9386861	9540790	13330484	-3943623	-3789694	42.0	39.7
2012-13	11461556	11915178	17574993	-6113437	-5659815	53.3	47.5
2013-14	12901809	13612370	20034526	-7132717	-6422156	55.3	47.2
2014-15	13363131	13883418	24469435	-11106304	-10586017	83.1	76.2

Source: AFS of Nileswaram Municipality from 2011-12 to 2014-15

Figure 1: Revenue Expenditure Gap Comparison at the Disaggregated Level



Source: AFS of Nileswaram Municipality from 2011-12 to 2014-15

Table 4: Growth Rate of Revenue, Expenditure and CR-CE Gap

Year	Growth Rate of Current Revenue	Growth Rate of Current Exp.	Growth Rate of CR-CE Gap
2011-12	-	-	-
2012-13	24.9	31.8	49.3
2013-14	14.2	14.0	13.5
2014-15	2.0	22.1	64.8

Source: AFS of Nileswaram Municipality from 2011-12 to 2014-15

Current Expenditure Gap was a little less at 76.2 per cent during 2014-15. It implies that approximately one Crore of rupees (around 80 per cent of own revenue mobilised) needs to be found by the municipality from other sources. This needs to be looked at seriously as it poses significant questions upon the financial conditions of municipality. Better inference may be drawn by looking at the growth rate. As it could be seen from table 4, the Current Revenue-Current Expenditure Gap has grown at a faster rate at 49.3 per cent from 2011-12 to 2012-13. This high growth rate of revenue-expenditure gap during the period was due to the faster growth of expenditure than the growth of revenue. While current revenue was registering a growth rate of 31.8 per cent, the growth rate of expenditure was at the rate of only 24.9 per cent. The gap has grown at a lower rate in the second period (2012-13 to 2013-14) as the growth rate of revenue and expenditure has fallen down during the period. The gap has again widened at a much faster rate of 64.8 per cent from 2013-14 to 2014-15 because of a drastic fall in revenue growth to just 2 per cent from 14.2 per cent and the expenditure has grown at a much faster rate of 22 per cent. The inference which comes out here is that the revenue has grown always at a lower rate than the growth rate of expenditure. Huge gap in revenue-expenditure was associated with very low rate of growth of revenue than the rate of growth of expenditure. Since cutting expenditure for a local body may not be possible keeping in mind the responsibilities they hold in the present democratic set up, raising revenue is the only viable option.

5. Financing Revenue-Expenditure Gap: The Role of General Purpose Fund

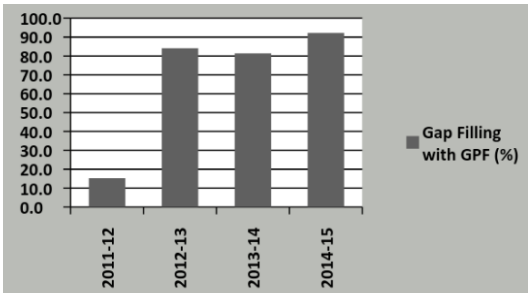
Many studies have established at various levels that the local bodies in general and urban local bodies in particular are generally facing severe revenue constraints. The analysis so far have also made it clear that the Nileswaram municipality has also been facing many revenue constraints. The revenue growth has been comparatively less than the growth rate of expenditure. The estimated gaps at various levels have also supported the argument that the fiscal position needs to be strengthened for the selected municipality as well. The analysis has come out with the finding that the municipality is finding the revenue for filling the gap from

Table 5: Gap Filling with GPF

Year	General Purpose Fund	CR-CE Gap	Gap Filling with GPF (%)
2011-12	24894594	-3789694	15.2
2012-13	6737448	-5659815	84.0
2013-14	7891229	-6422156	81.4
2014-15	11487000	-10586017	92.2

Source: AFS of Nileswaram Municipality from 2011-12 to 2014-15

Figure 2: Revenue Expenditure Gap Comparison at the Disaggregated Level



Source: AFS of Nileswaram Municipality from 2011-12 to 2014-15

the General Purpose Fund of revenue grants. The municipality is found to be increasingly depending upon the General Purpose Fund for financing the current revenue-current expenditure gap as it could be reflected from the table 5. While during 2011-12 only 15.2 per cent of the general purpose fund was utilised for filling the gap between current revenue and current expenditure, around 84 per cent was used for filling the gap during 2012-13. The proportion has further increased to 92.2 per cent during 2014-15. It comes out here that the municipality is filling the gap with the unconditional general purpose grant or rather adjusting with General Purpose Fund for carrying out its current expenditure. The utilisation from General Purpose Fund for carrying out its general administration expenditure (or rather filling the current revenue-current expenditure gap) is found to be increasing throughout the period as it could be seen from figure 2. Since the revenue expenditure gap has been filled mainly by the General Purpose Fund, it is interesting to look at how much of the expenditure in various periods is financed by the General Purpose Fund and how much by the current revenue component. It could be found from the table 6 that a significant proportion of current expenditure has been made with the assistance from General Purpose Fund. In 2011-12 around 28.4 per cent of the current expenditure has been financed by the General Purpose Fund. The share has gone up to 32.1 per cent during 2012-13 and subsequently to 43.3 per cent during 2014-15. This is due to the fact that the current expenditure is growing at a faster rate over the years than the current revenue; hence the municipality has to depend upon other sources like General Purpose Fund to meet its current expenditure. The picture is more glaring when the movement of share of General Purpose Fund is looked at. The share of General Purpose Fund in current expenditure is increasing over the years as it could be seen from the table 6. The municipality will be under severe fiscal stress if the General Purpose Fund does not increase much to meet the increasing expenditure requirements.

The growth of General Purpose Fund is highly significant in the coming years as far as Nileswaram municipality is concerned. Just to understand the financial position of the municipality with a futuristic perspective, the growth rate of revenue expenditure gap and General Purpose Fund may be compared. It could be observed that the growth rate of General Purpose Fund is well under the growth rate of Current Revenue-Current Expenditure Gap (refer to table 7). While General Purpose Fund has registered a negative growth rate (-72.9) from 2011-12 to 2012-13, the revenue-expenditure gap has grown at 49.3 per cent. The growth rate of revenue-expenditure gap has been higher than the growth rate of General Purpose Fund in all the periods reported except from 2012-13 to 2013-14. This becomes a serious concern for the municipality in the near future. It has to find mechanisms for rising own revenue. It has to think seriously about the possible ways of rising own revenue from property tax or profession tax or any other non-tax revenue sources.

Table 6: Growth of Revenue-Expenditure Gap and General Purpose Fund

Year	GPF	Growth Rate of GPF	CR-CE Gap	Growth Rate of CR-CE Gap
2011-12	24894594	-	3789694	-
2012-13	6737448	-72.9	5659815	49.3
2013-14	7891229	17.1	6422156	13.5
2014-15	11487000	45.6	10586017	64.8

Source: AFS of Nileswaram Municipality from 2011-12 to 2014-15

6. Components of Overall Revenue-Expenditure Gap

nalysing the components of overall revenue expenditure gap would serve the purpose of giving more clarifications to the analysis of revenue-expenditure gap in the previous sections. The overall revenue-expenditure gap or total surplus could be identified as the sum of surplus in General Purpose Fund and surplus in Revenue grants. The total surplus would vary from year to year according to the surplus or deficit in both General Purpose Fund and Revenue Grants. The overall revenue-expenditure gap was found to be surplus throughout all the years reported. The total surplus was approximately 2 Crore during 2011-12 and has come down to 35 Lakhs during 2012-13 and subsequently to 7 Lakhs during 2013-14. The surplus has gone up to 35 Lakhs during 2014-15. The huge surplus of more than 2 Crores during 2011-12 was mainly due to the huge surplus in General Purpose Fund. Around 91 per cent of the total surplus was contributed by the surplus in General Purpose Fund as could be found from the table 8. It is also surprising to see that only 8.9 per cent of the total surplus was contributed by the surplus in revenue grants.

The contribution of General Purpose Fund is not found to be stable as it came down drastically to 30.1 per cent during 2012-13; with revenue grants being 69.9 per cent. During 2013-14 the total surplus has been quite low because the surplus in General Purpose Fund has been reduced by the deficit in revenue grants. Hence, only in 2013-14 a part of General Purpose Fund was used for incurring programme expenditure or maintenance expenditure to be financed from revenue grants. In the year 2013-14 surplus from revenue grants have been significant as compared to the surplus in General Purpose Fund (22.9 per cent). Hence, the inference which can be derived here is that the total surplus or overall revenue-expenditure gap cannot be used as an index of financial health of the selected municipality as the surplus was mainly from revenue grants, which are conditional and tied up for the purpose; except during 2011-12 and 2013-14.

7. Concluding Observations

The present chapter attempts to estimate the revenue expenditure gap at various possible levels and locate the factors determining the gap by selecting the case of an emerging municipality in Kasaragod district of Kerala. It has been found that the overall revenue-expenditure gap was in surplus for all the years reported. By looking at the overall revenue-expenditure gap the fiscal position during the initial years was much better and the situation started worsening from 2012-13 onwards. The own revenue of Nileswaram municipality was found to be well short of current expenditure in all the years. The deficit was approximately one Crore for all the reported years and the gap was observed to be continuously growing. The current revenue was found to be growing always at a lower rate than the growth rate of current expenditure. A huge gap in current revenue-current expenditure was found for all the years and the gap was due to the very low growth rate of current revenue than the rate of growth of current expenditure. Apart from that, is has been found that the municipality is increasingly depending upon the General Purpose Fund for matching the current revenue-current expenditure gap. Hence, the municipality is somehow adjusting with General Purpose Fund for meeting most of its current expenditure. The share of General Purpose Fund for carrying out the general administration of the municipality is found to be increasing for all the periods reported. To make it more clearly, it was found that a significant proportion of current expenditure has been made with the assistance from General Purpose Fund. The share of General Purpose Fund in current expenditure is increasing over the years. The municipality will be under severe fiscal stress if either the General Purpose Fund does not increase much to meet

Table 7: Components of Total Surplus

Year	Surplus / Deficit in General Purpose Fund	Surplus / Deficit in Revenue Grants	Total Surplus	S/D in GPF (%)	S/D in RG (%)
2011-12	21104900	2056775	23161675	91.1	8.9
2012-13	1077633	2499447	3577080	30.1	69.97
2013-14	1469073	-717355	751718	195.4	-95.4
2014-15	900983	3031978	3932961	22.9	77.1

Source: AFS of Nileswaram Municipality from 2011-12 to 2014-15

the increasing expenditure requirements of the municipality. It is also found that the growth rate of General Purpose Fund is well under the growth rate of Current Revenue-Current Expenditure Gap. This needs to be looked at seriously by the municipality in the coming years.

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Hyper Branched Polycarbosilanes: Synthesis And Characterization

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Abstract

Polycarbosilanes were synthesized by Wurtz-type coupling of monomer/s in the presence of sodium metal. The polycarbosilanes were characterised using various analytical techniques. All the polycarbosilanes are of highly cross-linked structure and amorphous in nature. All of them possess high temperature stability, high surface area and is hydrophobic in nature.

1. Introduction

Organosilicon chemistry lies at a point intermediate between organic and inorganic chemistry and draws liberally from both fields. As polymers that contain both Si and C in their backbone structure, polycarbosilanes have been of particular interest as precursors to silicon carbide, an important high temperature structural ceramic and semiconductor material. These polymers have been of primary interest as potential precursors to silicon carbide, a ceramic material which is of considerable technological importance due to its high thermal, chemical and oxidative stability, as well as for its electrical (semiconductor) and mechanical (high hardness, shock resistance, strength etc) properties.

Polycarbosilanes show a variety of potential applications that are opening up for these polymers in areas such as nanotechnology [1], surface science, catalysis [2-6], liquid crystals [7-9], new ceramic materials [10-14], organic-inorganic hybrids etc. PCS can be prepared in one step from simple silane monomers or mixtures containing vinyl, halo or halomethyl moieties in the presence of an active metal. The participation of vinyl groups produces partial branching. The Wurtz type coupling reaction, which employs alkali metals, can be used for the preparation of PCS. It generally leads to extensive crosslinking and appreciable Si-Si and C-C bonding in addition to Si-C bonding [15-17]. The highly crosslinked nature of the PCS provides scope for high surface area for the polymer [4,5]. High surface area is one of the main criteria for materials which are used as solid supports in heterogeneous catalysis.

In most cases, these are "one-pot" reactions that employ monomers that bear halogen or alkoxy groups on the C and Si end of the unit. Indeed, hyper branched polycarbosilanes have been described, in general, as "obtained in one synthetic step via, random, one-pot polymerization of multifunctional monomers of ABn type".

The product of these reactions generally consists of a polydisperse mixture of oligomers and polymers that are coupled primarily through Si-C bonds, but which can also contain appreciable Si-Si bonding.

Polymers of trichloromethylsilane and Diethoxymethylvinylsilane, copolymers of

trichloromethylsilane with vinyl silanes such as chlorodimethyl vinyl silane, diethoxymethylvinyl silane and vinyltrimethoxy silane were prepared.

2. Experimental

2.1. Materials

Monomers, Methyltrichlorosilane, Chlorodimethylvinylsilane, Diethoxymethylvinylsilane and Vinyltrimethoxysilane were purchased from Sigma-Aldrich, USA. All the other chemicals were purchased from different local suppliers and used as received. All the solvents were purified according to the standard procedure. All other fine chemicals were used as received.

2.2. Instrumental Techniques

FTIR spectra were recorded on JASCO model 4100 FTIR spectrometer as KBr pellets. Solid state ^{29}Si-CP-MAS NMR in 300 MHz Bruker DSX Spectrometer (NMR research centre, IISc. Bangalore). TG-DTA analysis was done on Perkin-Elmer Diamond model TG/DTA system using Platinum as standard. Samples were heated under nitrogen atmosphere from 50^{o}C- 1000^{o}C at a rate of 20^{o}C/min. The X- Ray Diffraction analysis was carried out using Rigaku X-ray Photometer. The sample was scanned over the range of 20^o-90^o angles with an increment of 0.05^o angle and with the rotation speed 5^o/min. Surface area and porosity were measured using Tristar-3000V6.07A of Micromeritics at Sud-chemie-Cochin.

2.3. Characterization

The polymers were characterized by FT- IR spectroscopy, ^{29}Si-CP-MAS NMR, X- ray diffraction analysis and TG-DTA methods.

FT- IR spectra give the peaks corresponding to the Si-C and C-H vibrations. That are the main bonds present in a polycarbosilane.

^{29}Si NMR is probably the most significant method for the silicon analysis. The major problem associated with ^{29}Si spectroscopy - poor sensitivity because of low isotopic abundance, long relaxation times- appear to have been solved by the use of shiftless relaxation reagents in combination with heteronuclear decoupling and Fourier Transform spectroscopy [18]. From ^{29}Si CP-MAS NMR spectra of polymers we will get a very clear idea about the structure of the polymers.

X- ray diffraction plots give an idea about the morphology of the polymer. Morphological nature of the polycarbosilanes was confirmed from XRD results and the thermal stability of the prepared polymers was determined by TG-DTA method.

Surface area analysis was carried out by N_2 gas adsorption methods.

2.4. Procedure for polymer preparation

Five polycarbosilanes were prepared by Wurtz-type coupling of monomers in the presence of sodium metal and is named as PCS 1 to 5.

2.4.1. Preparation of PCS 1

Preparation of PCS 1 was prepared conducted by a two-step reaction. First, in which a polysilane is prepared and then by the thermolysis of polysilane to polycarbosilane. Polymer with methyltrichlorosilane as monomer was prepared initially. The Sodium metal (1.2g, 54.375mmol) in anhydrous toluene (50mL) was refluxed and stirred vigorously to make dispersion. Methyltrichlorosilane (6.4mL, 54.375mmol) was added drop wise. The reaction was highly exothermic. After the addition of monomers, the reaction mixture was refluxed for 12h. The whole process was carried out under Nitrogen atmosphere. The reaction mixture was cooled and the suspension obtained was filtered, and washed initially with methanol and then with water in a Soxhlet extractor for 24 h and dried under vacuum.

Thermolysis of polysilane

The polysilane was heated upto 400^{o}C in a muffle furnace for 4h. The polysilane has undergone Kumada rearrangement and gave crosslinked polycarbosilane.

2.4.2. Preparation of PCS 2

Polymer of Methyltrichlorosilane and chlorodimethylvinylsilane was prepared. In a 250mL round bottom flask, the Sodium metal (2.5g, 108.75mmol) in anhydrous toluene (50mL) was refluxed and stirred vigorously to make dispersion. A mixture of methyl trichlorosilane (6.4mL, 54.375mmol) and chlorodimethylvinylsilane (7.4mL, 54.375mmol) was added drop wise.

The reaction was highly exothermic. After the addition of monomers, the reaction mixture was refluxed for 12h. The whole process was carried out under Nitrogen atmosphere. It was cooled and the obtained suspension was filtered, washed with methanol, water and dried under vacuum.

2.4.3. Preparation of PCS 3

Polymer of Methyltrichlorosilane and diethoxymethylvinyl silane was prepared. In a 250mL round bottom flask, the Sodium metal (2.5g, 108.75mmol) in anhydrous toluene (50mL) was refluxed and stirred vigorously to make dispersion. A mixture of methyl trichlorosilane (6.4mL, 54.375mmol) and diethoxymethylvinylsilane (10.24 mL, 54.375mmol) was added drop wise. The reaction was highly exothermic. After the addition of monomers, the reaction mixture was refluxed for 12h. It was cooled and the obtained suspension was filtered, washed with methanol, water and dried under vacuum. The whole process was carried out under Nitrogen atmosphere.

2.4.4. Preparation of PCS 4

Polymer of Methyltrichlorosilane and vinyltrimethoxy silane was prepared. In a 250mL round bottom flask, the Sodium metal (2.5g, 108.75mmol) in anhydrous toluene (50mL) was refluxed and stirred vigorously to make dispersion. A mixture of methyl trichlorosilane (6.4mL, 54.375mmol) and trimethoxyvinylsilane (8.13mL, 54.375mmol) was added drop wise. The reaction was highly exothermic. After the addition of monomers, the reaction mixture was refluxed for 12h. It was cooled and the obtained suspension was filtered, washed with methanol, water and dried under vacuum. The whole process was carried out under Nitrogen atmosphere.

2.4.5. Preparation of PCS 5

Polymer using Diethoxymethylvinylsilane as monomer was prepared. The Sodium metal (1.2g, 54.375mmol) in anhydrous toluene (50mL) was refluxed and stirred vigorously to make dispersion. Diethoxymethylvinylsilane (10.24 mL, 54.375mmol) was added drop wise. The reaction was highly exothermic. After the addition of monomers, the reaction mixture was refluxed for 12h. The whole process was carried out under Nitrogen atmosphere. The reaction mixture was cooled and the suspension obtained was filtered, and washed initially with methanol and then with water in a Soxhlet extractor for 24h and dried under vacuum.

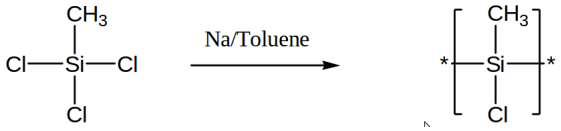
3. Result and Discussion

3.1. General procedure for polymerization

The polycondensation of monomers to prepare polycarbosilanes is adopted. The monomers on treatment with sodium metal resulted in the polymer, by Wurtz-type coupling. The Sodium metal and monomer were taken in 1:1 ratio. For copolymerization the monomer ratio was 1:1. The polymerizations were carried out in dry toluene. The sodium metal was made into dispersion in toluene and the monomer was added. The reaction is highly exothermic. After the addition, the reaction mixture was refluxed for 12hrs. The reaction was carried out under N2 atmosphere. It was filtered and washed in a soxhlet with water and dried. The yield of the prepared polymers was estimated and the chlorine content was determined using the modified Volhards method [4,5,19].

3.2. PCS1

Preparation of PCS 1 was prepared conducted by a two-step reaction. First, in which a polysilane is prepared and then by the thermolysis of polysilane to polycarbosilane. In the initial step polysilane was prepared from trichloromethylsilane. (Scheme: 1)



Scheme 1: Preparation of **PMS**

The FT-IR spectrum of the polymer has the corresponding peaks of the polysilane. The spectrum shows a peak at 2952 cm^{-1} which is corresponding to the C-H stretching, peak at 1447 cm^{-1} is due to Si-CH_3 stretching, peak at 1252 cm^{-1} corresponds to Si-CH_3 deformation and 777 cm^{-1} is due to Si-CH_3 wagging. 1045 cm^{-1} due to the Si-O stretching.

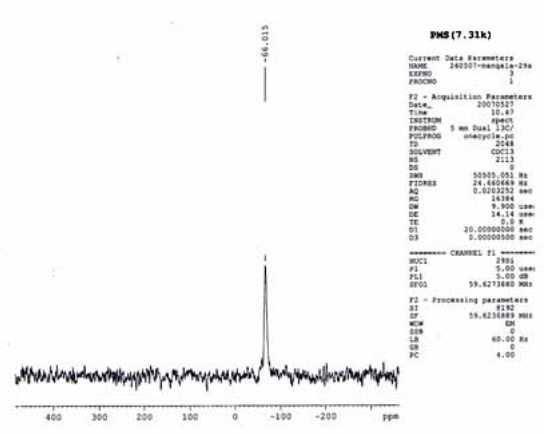


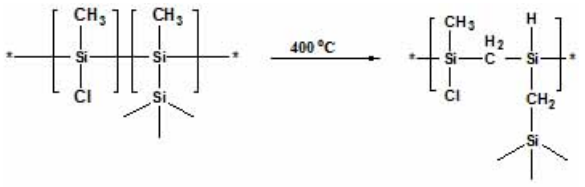
Figure 1: ^{29}Si NMR spectra of PMS

^{29}Si NMR spectrum of PMS is shown in Figure 1. Chemical shifts are based on tetramethylsilane as the internal standard. In the spectrum, the peak around -66.01 ppm was assigned to be due to the resonance of Silicon in $(CH_3)SiSi$ unit (branching unit). A small peak in close proximity to it around -60 ppm is corresponding to the $(CH_3)ClSiSi_2$ unit (linear unit). The branching unit was formed by the elimination of Cl atom from the Si-Cl groups during Wurtz coupling.

Thermolysis of polysilane

The polysilane was heated upto 400^{o}C in a muffle furnace for 4h. The polysilane has undergone Kumada rearrangement and gave crosslinked polycarbosilane (Scheme: 2).

The polycarbosilane was characterised with TG-DTA, X-ray Diffraction analysis, Surface area analysis, FT-IR spectroscopy and ^{29}Si NMR spectroscopy.



Scheme 2: Preparation of PCS 1

The FT-IR spectrum of PCS 1 has the corresponding peaks of the polycarbosilane. Peaks at 1441, 1253, 772 cm^{-1} are clearly seen in the spectrum which corresponds to the $CHSiCH_3$ and SiC. The spectrum shows peaks at 2948 and 2899 cm^{-1} which corresponds to the C-H stretching and 1038 cm^{-1} is due to CH_2 wagging in Si-CH_2-Si bond.

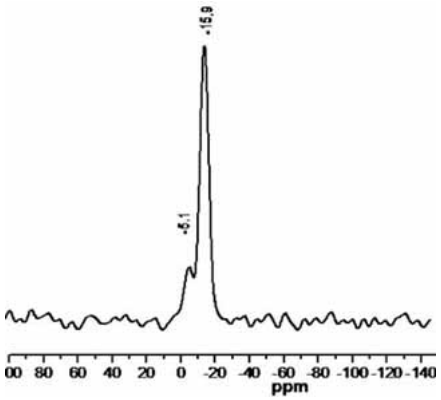
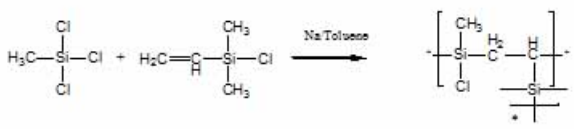


Figure 2: ^{29}Si NMR spectrum of PCS 1

^{29}Si CP-MAS NMR spectrum of the PCS 1 is shown in Figure 2. Spectrum gives a clear idea about the structure of the polycarbosilane. The spectrum shows a single peak. Peak around -15ppm corresponds to $CH_3Si(CH_2)_2$ linkage. The broad signal centered at -15.9 ppm is attributed to the overlapped signals of $(CH_2)_2Si(CH_2)_2$ units also which have a resonance around -5.1 ppm.

3.3. PCS 2

Polymer of Methyltrichlorosilane and chlorodimethylvinylsilane was prepared (Scheme: 3).



Scheme 3: Preparation of PCS 2

The FT-IR spectra of the polymers have the corresponding peaks of the polycarbosilane. Peak at 1514, 1231, 800 cm^{-1} are clearly seen in the spectra

which correspond to the $CHSiCH_3$ and SiC bonding. The FT-IR spectrum of PCS 2 has the corresponding peaks of the polycarbosilane. Peaks at 1514, 1231, 800 cm^{-1} are clearly seen in the spectrum which correspond to the $CHSiCH_3$ and SiC. The spectrum shows a peak at 2828 cm^{-1} which corresponds to the C-H stretching, peak at 1514 cm^{-1} which is due to Si-CH_3 stretching, 1231 cm^{-1} corresponding to Si-CH_3 deformation, 1038 cm^{-1} due to CH_2 wagging in Si-CH_2-Si bond and 800 cm^{-1} due to Si-CH_3 wagging.

The peaks assigned to Si-O-Si bonds (around 1040 cm^{-1}) are observed in all polymers, since this peak intensified with prolonged air exposure, polymers may have oxidised during the handling and recording processes.

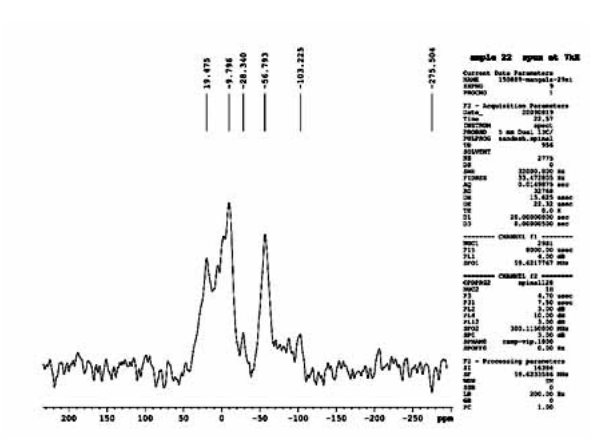
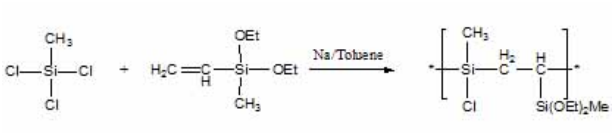


Figure 3: ^{29}Si CP-MAS NMR spectra of PCS 2

The ^{29}Si CP-MAS NMR spectra of PCS 2 is shown in Figure 2. Peak around 19.475 ppm and -56.793 ppm corresponds to Si-O-Si and Si-O-C linkage respectively, which may be resulted due to the oxidation of the pendant group. A peak at -9.796 ppm which corresponds to $(CH_3)_2Si(CH_2)_2$ may might be formed due to the side chain polymerization, peak corresponds to $(CH_2)_2SiSi$ -28.340 ppm which might be formed by the head to head polymerization of the monomers.

3.4. PCS 3

Polymer of Methyltrichlorosilane and diethoxymethylvinyl silane was prepared (Scheme: 4) FT-IR spectrum of PCS 3 exhibited peaks



Scheme 4: Preparation of PCS 3

at 2967 cm^{-1} corresponding to C-H stretching in Si-CH_3, 1642 and 1414 cm^{-1} due to C-H stretching or CH_2 deformation in Si-CH_3, 1258 cm^{-1} to Si-CH_3 deformation, 1076 cm^{-1} to CH_2 wagging in Si-CH_2-Si, 777 cm^{-1} due to Si-CH_3 wagging and Si-C stretching.

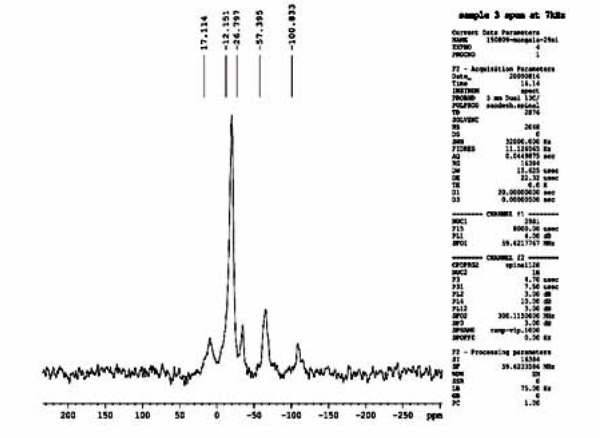
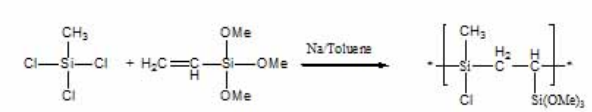


Figure 4: ^{29}Si CP-MAS NMR spectra of PCS 3

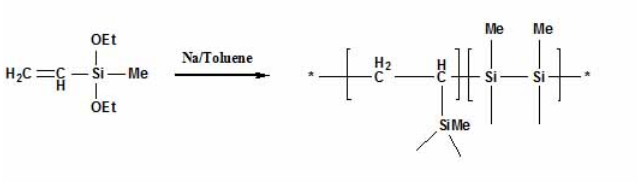
^{29}Si CP-MAS NMR spectra of PCS 3 is in Figure 4. Peak at 17.114 ppm corresponds to Si-O-Si bond, which would be resulted by the oxidation of polymer, -12.151ppm peak corresponds to $(CH_3)Si(CH_2)_2$ linkage, -26.797 ppm $(CH_2)_2Si$ Si bond which had resulted by the head to head polymerization, peak around-57.395 ppm corresponds to the $Si(OEt)_2$ in the side chain.

3.5. PCS 4

Polymer of Methyltrichlorosilane and vinyltrimethoxy silane was prepared (Scheme: 5). The FT-IR spectrum of the polymer has the corresponding peaks of the polycarbosilane. Peak at 1453, 1270, 1082, 783 cm^{-1} are clearly



Scheme 5: Preparation of PCS 4



Scheme 6: Preparation of PCS 5

seen in the spectrum which corresponded to the $CHSiCH_3$ and SiC. The spectrum showed a peak at 2929cm^{-1} which was corresponding to the C-H stretching, 1635, 1453 cm^{-1} due to Si-CH_3 stretching, 1270 cm^{-1} corresponding to Si-CH_3 deformation, 1082 cm^{-1} due to CH_2 wagging in Si-CH_2-Si bond and 783 cm^{-1} due to Si-CH_3 wagging.

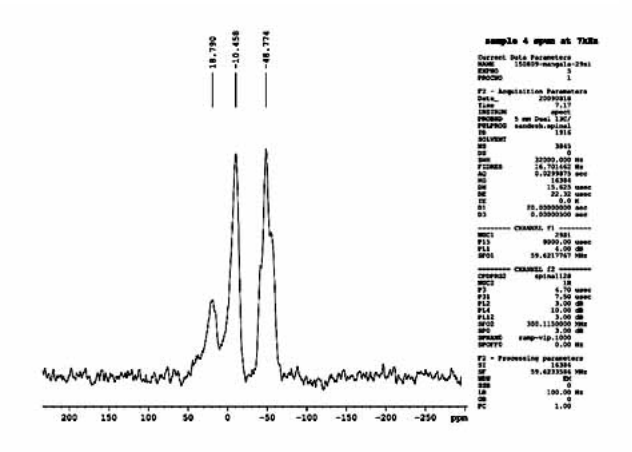


Figure 5: ^{29}Si CP-MAS NMR spectra of PCS 4

^{29}Si CP-MAS NMR spectra of PCS 4 is in Figure 5. Peak around 18.790 ppm corresponds to Si-O-Si linkage formed by the oxidation of the polymer, peak at -10.458 ppm corresponds to $CH_3Si(CH_2)_2$ linkage, -48.774 ppm peak is of pendant $CHSi(OCH_3)_3$.

3.6. PCS 5

Polymer using Diethoxymethylvinylsilane as monomer was prepared (Scheme: 6). FT-IR spectrum of PCS 5 has the peaks at 2961 cm^{-1} corresponding to C-H stretching in Si-CH_3, 1647 and 1407 cm^{-1} due to C-H stretching or CH_2 deformation in Si-CH_3, 1264 cm^{-1} due to Si-CH_3 deformation, 1095 cm^{-1} due to CH_2 wagging in Si-CH_2-Si, 790 cm^{-1} due to Si-CH_3 wagging and Si-C stretching.

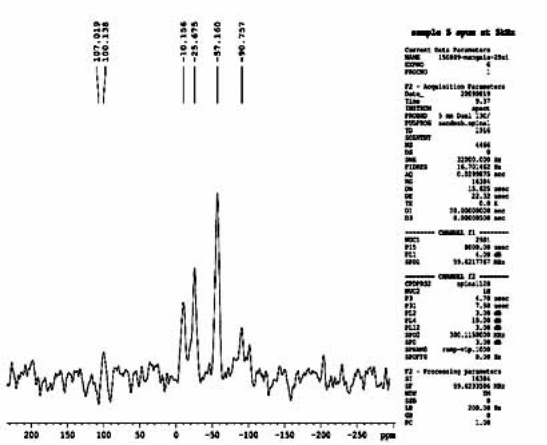


Figure 6: ^{29}Si CP-MAS NMR spectra of PCS 5

In ^{29}Si CP-MAS NMR spectra of PCS 5, Chemical shift of -10.156 ppm is corresponding to $(CH_3)2Si(CH_2)_2$ linkage, -25.675 ppm peak is of the CH_2SiCH_2 bond, a peak around -57.160 ppm is an evidence for $CHSi(OCH_3)_3$.

Halide Estimation

The quantitative chlorine estimation of the polycarbosilanes was done by modified Volhard's method. The chlorine estimation was done for all the synthesized polycarbosilanes except PCS 5. All other polycarbosilanes gave expected results for the estimation and the results are listed in Table 1.

Polymer	PCS 1	PCS 2	PCS 3	PCS 4
Chlorine (mmol/g)	3.9	5.1	4.5	4.1

Table 1: Chlorine estimation of polycarbosilanes

4. Thermal properties of polycarbosilanes

Inorder to investigate the thermal behaviour of the polycarbosilanes; the TG-DTA analysis was done. The polycarbosilanes were subjected to temperature scan from 50 to 1000^{o}C in N_2 atmosphere at a rate of 20^{o}C/min. The result is illustrated in Figure 7.

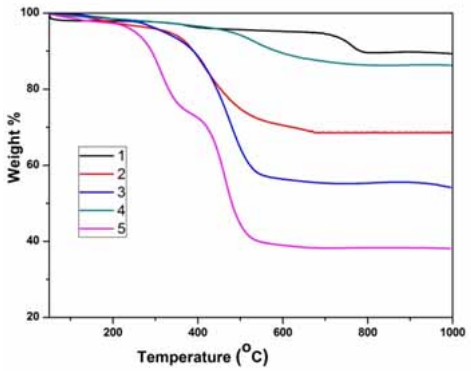


Figure 7: Thermogravimetric analysis of polycarbosilanes

In the TGA curve, 3% weight loss below 300^{o}C is attributed to the vaporization of low molecular weight oligomers. From 300-800^{o}C the weight loss is due to the release of gaseous products such as methane and hydrogen during the conversion of the Si-Si bond to Si-C bond. No obvious weight loss was observed in the range of 800-1000^{o}C. High temperature stability is attributed to substantial cross-linking in PCS.

he TGA residues of the polycarbosilanes are given in Table 2. From the TGA curves it is clear that out of the five polycarbosilanes, PCS 1 and PCS 4 have highly cross-linked structure. PCS 1, polycarbosilane prepared by the thermolysis of polysilane gave high char yield. It has low volatile substituents because it had under gone thermolysis prior to analysis. During thermolysis the low volatile fractions might have undergone evaporation and hence high char yield. Polycarbosilane prepared by the homopolymerization of

Entry	TGA residue at 1000^{o}C (%)
PCS 1	89
PCS 2	68
PCS 3	54
PCS 4	86
PCS 5	38

Table 2: TGA residue of the polymers

diethoxymethylvinylsilane (PCS 5) have low char yield. This may be because of the less cross-linked nature of the polymer. Other polymers PCS2 and PCS3 gave average char yield. The TGA curves suggesting that PCS 4 have great potential to be promising precursor to SiC.

4.1. X-ray diffraction analysis

X-ray diffraction analysis of all the prepared polycarbosilanes was performed. The sample was scanned over the range of 20^o -90^o angles with an increment of 0.05^o angle and with the rotation speed of 5^o/min. (Figure: 8)

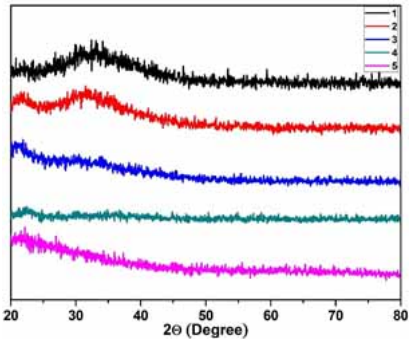


Figure 8: X-Ray Diffraction pattern of prepared polycarbosilanes

4.2. Surface Area Analysis

Nitrogen gas adsorption methods are commonly employed to determine surface area and pore size distribution of solids. The polycarbosilanes are subjected to surface area analysis by BET surface area analyzer. Results are tabulated (Table 3).

Sample	Surface area (m^2/g)	
	BET	**Langmuir**
PCS 1	333.3	506.2
PCS 2	229.4	458.5
PCS 3	203.6	397.8
PCS 4	679.3	1171.2
PCS 5	154.2	275.2

Table 3: TGA residue of the polymers

The high surface area of PCS 4 is attributed to its high cross-linked structure. And PCS 4 has mesoporous nature also. The thermogravimetric analysis results are in good agreement with these results. From the surface area results it is clear that the homopolymer prepared from diethoxymethylvinylsilane has low surface area which can be explained because of its less cross-linked nature.

The synthesized polycarbosilanes shows hydrophobic nature. All of them are not wetted by water.

5. Conclusion

Preparation of five polycarbosilanes was done successfully. Halogen estimation was done by Volhard's method. The polycarbosilanes were characterised using various analytical techniques such as TG-DTA, X-ray diffraction analysis, FT-IR, ^{29}Si NMR spectroscopies and surface area analysis. From the results of analytical data, the structure of polycarbosilanes was deducted. All the polycarbosilanes are of highly cross-linked structure and amorphous in nature. All of them possess high temperature stability and high surface area. All of them shows hydrophobicity. The polycarbosilanes, except PCS 5, are capable of further functionalization. The functionalization of polycarbosilanes can be used to make use of the polycarbosilanes in a variety of applications.

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Analysis of a Discrete Time Bulk Service Queue

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Abstract

In this paper we consider a discrete time (a, c, d) policy bulk service queueing system with or without accessibility to the ongoing service batch under customer's choice. Here the server begins service only when the number of customers in the queue (n), is at least c and serves a maximum of d customers in a batch with or without accessibility to the batches of ongoing service, if the batch size is less than b under the customer's choice. That is, the arriving customer decides to join the ongoing service (accessible batch) with probability p or waits till the next service batch starts. The server continue to serve even when the system size is less than c but not less than a secondary limit $a(a \leq c \leq b \leq d)$, after a service completion epoch. The analysis of the model is considered and the steady state probabilities, expected queue length, expected cost function are obtained and the associated optimality problem is also discussed.

Keywords: Discrete time queues, (a, c, d) policy, Bulk service queue, Single server, Accessible batches, Customer's choice, Steady state probabilities, Expected cost function.

1. Introduction

Batch service queueing models have potential applications in many areas, for example, in the loading and unloading of cargos at a seaport, in traffic signal systems, in communication systems (each message consisting of different packets), in transportation systems (buses, guided tours, medical evaluation systems, etc.), in manufacturing systems(electrical, electronic and mechanical industries making products such as cars, computers, machines, etc., where each job is considered as the accumulation of different tasks), in a computer network where jobs are processed in batches. In typical batch service queueing models, once the service is started, arriving customer cannot enter the service station though enough space is available to accommodate it. But in many practical situations the arriving customers will be considered for service with current batch in service with some limitation, for example cinema hall, elevator, etc. It may be noted that the accessible batch service has more economic utilizations in providing better service to the queue. For example, in many shuttle transportation systems, we observe units being transported according to accessibility rule with some limitation.

Queueing systems with accessible and non accessible batches were considered by many researchers in the past (see for example: Kleinrock(1975), Chiamsiri and leonard(1981), Sivasamy(1990), Gross and Harris(1999), Baburaj and Manoharan(1999), Baburaj(2000), Goswami and Samanta(2009) etc.).

Baburaj and Surendranath (2006) considered an (a, c, d) policy $M/M/1$ bulk service queue with accessible and non accessible batches. In the model considered by Baburaj and Surendranath (2005), the server starts service only when the queue size is atleast c and continue to serve even when the system size is less than c but not less than a secondary limit $a(a \leq c)$, after a service completion epoch, such a rule is called Modified General Bulk Service Rule (Modified GBSR). Baburaj and Surendranath (2006) further assumed that the server allows the late entries to join a batch in course of ongoing service till the batch size is less than the accessible limit $b(a \leq c \leq b \leq d)$.

Here a serious problem arises concerning the service of the accessible batches,where a customer arrives when the service of an accessible batch is going on. In certain cases, the arriving customer may not be interested in joining the on going service batch, which commenced the service before it's arrival, since it cannot get full service from that service batch. So the arriving customer has a chance either to join the ongoing service (accessible batch) or to wait in the queue till the service of the next batch begins.

In this model, we consider an (a, c, d) policy bulk service queue with accessible and non-accessible batches under the customer's choice. Here the customers arrive according to a poisson process with parameter λ. The server begins service if there are atleast c customers in the queue and allows late arrivals to join the ongoing service till the batch size is less than b (accessible batch)and the arriving customer has a choice either to enter the accessible batch or to wait in the queue till the service of the next batch begins. We assume that the arriving customer join the accessible batch with probability p or with probability $1 - p$ he waits in the queue till the service of the next batch begins. Here the server continue to serve even when the queue size n is less than c but not less than a secondary limit a $(a \leq c \leq b \leq d)$, after a service completion epoch. The service time distribution is assumed to be exponential with parameter μ.

2. Description of the model

- Let $V_n(t-)$, $n = 0, 1, 2, ..., c-1$ denote the probability that the server is idle and there are n units waiting in the queue.

- $\Pi_{1n}(t-)$, $n = a, a+1, a+2, ..., b-1$ denote the probability that the server is busy with an accessible batch and there are n units in the queue.

- $\Pi_{2n}(t-)$, $n = 0, 1, 2, ...$ denote the probability that the server is busy with a non accessible batch and there are n units in the queue (excluding those in service).

- Here the arriving unit may find the system in any of the following cases.

    1. (0,n),$(n = 0, 1, 2, ..., c-2)$ in this case the system is idle.
    2. (0,c-1),in this case the state of transition will be (1,0).
    3. (1,n),$(n = a, a+1, a+2, ..., b-2)$ in this case with probability p the state of transition will be(1,n+1) and with probability $1 - p$ the state of transition will be (2,1).
    4. (1,b-1),in this case with probability p the system enters to the new state (2,0) and with probability $1 - p$ the new state of transition will be (2,1).
    5. (2,n),n=0,1,2,.... the state of transition will be (2,n+1).

3. Analysis of the Model

Here the state space of the system is

$$\begin{aligned} S &= S_1 \cup S_2 \cup S_3, \\ \text{where} \quad S_1 &= \{(0,n), n = 0,1,...,c-1\}, \\ S_2 &= \{(1,n), n = a, a+1, a+2, ..., b-1\} \\ \text{and } S_3 &= \{(2,n), n = 0,1,2,...\} \end{aligned}$$

Hence the forward equations governing the transitions are

$$V_0(t+1) = (1-\lambda)V_0(t) + \mu(1-\lambda)\sum_{n=a}^{b-1}\Pi_{1n}(t) + \mu(1-\lambda)\Pi_{20}(t) \tag{1}$$

$$V_n(t+1) = (1-\lambda)V_n(t) + \lambda V_{n-1}(t) + \mu(1-\lambda)\Pi_{2n}(t), \quad n = 1,2,...,a-1 \tag{2}$$

$$V_n(t+1) = (1-\lambda)V_n(t) + \lambda V_{n-1}(t), \quad n = a, a+1, ..., c-1 \tag{3}$$

$$\Pi_{1a}(t+1) = (1-\lambda)(1-\mu)\Pi_{1a}(t) + \mu(1-\lambda)\Pi_{2a}(t) \tag{4}$$

$$\Pi_{1n}(t+1) = (1-\lambda)(1-\mu)\Pi_{1n}(t) + p\lambda(1-\mu)\Pi_{1n-1}(t) + \mu(1-\lambda)\Pi_{2n}(t), \quad n = a+1, a+2, ..., c-1 \tag{5}$$

$$\Pi_{1c}(t+1) = (1-\lambda)(1-\mu)\Pi_{1c}(t) + p\lambda(1-\mu)\Pi_{1c-1}(t) + \lambda(1-\mu)V_{c-1}(t) + \mu(1-\lambda)\Pi_{2c}(t) \tag{6}$$

$$\Pi_{1n}(t+1) = (1-\lambda)(1-\mu)\Pi_{1n}(t) + p\lambda(1-\mu)\Pi_{1n-1}(t) + \mu(1-\lambda)\Pi_{2n}(t), \quad n = c+1, c+2, ..., b-1 \tag{7}$$

$$\Pi_{20}(t+1) = (1-\lambda)(1-\mu)\Pi_{20}(t) + \mu(1-\lambda)\sum_{n=b}^{d}\Pi_{2n}(t) + p\lambda(1-\mu)\Pi_{1b-1}(t) \tag{8}$$

$$\Pi_{21}(t+1) = (1-\lambda)(1-\mu)\Pi_{21}(t) + \lambda(1-\mu)\Pi_{20}(t) + (1-p)\lambda(1-\mu)\sum_{n=a}^{b-1}\Pi_{1n}(t) + \mu(1-\lambda)\Pi_{2d+1}(t) \tag{9}$$

$$\Pi_{2n}(t+1) = (1-\lambda)(1-\mu)\Pi_{2n}(t) + \lambda(1-\mu)\Pi_{2n-1}(t) + \mu(1-\lambda)\Pi_{2n+d}(t), \quad n = 2,3,... \tag{10}$$

3.1. Steady State Probabilities

The steady state probabilities of the system states are given by

$$\begin{aligned} V_n &= \lim_{t\to\infty} V_n(t), n = 0,1,...,a-1 \text{ and} \\ \Pi_{1n} &= \lim_{t\to\infty} \Pi_{1n}(t), n = a, a+1, a+2, ..., c-1 \\ \Pi_{2n} &= \lim_{t\to\infty} \Pi_{2n}(t), n = 0,1,2,... \end{aligned}$$

From (3.1) to (3.10) the equations for the steady state probabilities are given by,

$$\lambda V_0 = \mu(1-\lambda)\sum_{n=a}^{b-1}\Pi_{1n} + \mu(1-\lambda)\Pi_{20} \tag{11}$$

$$\lambda V_n = \lambda V_{n-1} + \mu(1-\lambda)\Pi_{2n} \quad , \quad n = 1, 2, ..., a-1 \tag{12}$$

$$\lambda V_n = \lambda V_{n-1}, \qquad n = a, a+1, ..., c-1 \tag{13}$$

$$(\lambda + \mu - \lambda\mu)\Pi_{1a} = \mu(1-\lambda)\Pi_{2a} \tag{14}$$

$$(\lambda + \mu - \lambda\mu)\Pi_{1n} = p\lambda(1-\mu)\Pi_{1n-1} + \mu(1-\lambda)\Pi_{2n}, n = a+1, a+2, ..., c-1 \tag{15}$$

$$(\lambda + \mu - \lambda\mu)\Pi_{1c} = p\lambda(1-\mu)\Pi_{1c-1} + \lambda(1-\mu)V_{c-1} + \mu(1-\lambda)\Pi_{2c} \tag{16}$$

$$(\lambda + \mu - \lambda\mu)\Pi_{1n} = p\lambda(1-\mu)\Pi_{1n-1} + \mu(1-\lambda)\Pi_{2n}, n = c+1, c+2, ..., b-1 \tag{17}$$

$$(\lambda + \mu - \lambda\mu)\Pi_{20} = \mu(1-\lambda)\sum_{n=b}^{d}\Pi_{2n} + p\lambda(1-\mu)\Pi_{1b-1} \tag{18}$$

$$(\lambda + \mu - \lambda\mu)\Pi_{21} = \lambda(1-\mu)\Pi_{20} + (1-p)\lambda(1-\mu)\sum_{n=c}^{b-1}\Pi_{1n} + \mu(1-\lambda)\Pi_{2d+1} \tag{19}$$

$$(\lambda + \mu - \lambda\mu)\Pi_{2n} = \lambda(1-\mu)\Pi_{2n-1} + \mu(1-\lambda)\Pi_{2n+d}, \qquad n = 2, 3, ... \tag{20}$$

Equation (3.20)can be written as

$$\mu(1-\lambda)z^{d+1} - (\lambda + \mu - \lambda\mu)z + \lambda(1-\mu) = 0 \tag{21}$$

Using Rouche's theorem there exist a unique positive real root less than unity of the equation (3.21) such that

$$\Pi_{2n} = \Pi_{21}r^{n-1}, \quad n = 2, 3, ...$$

From (3.17)

$$\begin{aligned}\Pi_{1n} &= pe_1\Pi_{1n-1} + e_2\Pi_{2n} \\ &= \Pi_{1c}(pe_1)^{n-c} + \Pi_{21}e_2r^c\left[\frac{(pe_1)^{n-c} - r^{n-c}}{pe_1 - r}\right]; \quad n = c+1, c+2, ..., b-1 \\ &\text{where } e_1 = \frac{\lambda(1-\mu)}{\lambda + \mu - \lambda\mu}; \quad e_2 = \frac{\mu(1-\lambda)}{\lambda + \mu - \lambda\mu}\end{aligned}$$

From (3.15)

$$\Pi_{1n} = \Pi_{1a}(pe_1)^{n-a} + \Pi_{21}e_2r^a\left[\frac{(pe_1)^{n-a} - r^{n-a}}{pe_1 - r}\right]; \quad n = a+1, a+2, ..., c-1$$

From (3.14)

$$\begin{aligned}\Pi_{1a} &= e_2\Pi_{2a} \\ &= e_2\Pi_{21}r^{a-1} \\ \therefore \Pi_{1n} &= \Pi_{21}e_2r^{a-1}\left[(pe_1)^{n-a} + \frac{r}{pe_1 - r}\left((pe_1)^{n-a} - r^{n-a}\right)\right], n = a+1, a+2, ..., c-1\end{aligned}$$

Using (3.16) and (3.19) ,

$$\Pi_{1c} = e_1\left(p\Pi_{1a-1} + V_{c-1}\right) + e_2\Pi_{2c}$$

From (3.11) to (3.18) we get,

$$\Pi_{20} = \Pi_{21}D_1 + \Pi_{1c}D_2$$

where, $D_1 = e_2\left[\dfrac{r^{b-1} - r^d}{1-r} + pe_1r^c\left(\dfrac{(pe_1)^{b-1-c} - r^{b-1-c}}{pe_1 - r}\right)\right]$,

$D_2 = (pe_1)^{b-c}$

$$\begin{aligned} V_0 &= \Pi_{21}D_3 + \Pi_{1c}D_4 \\ V_n &= \Pi_{21}\left[D_3 + e_3\frac{1-r^n}{1-r}\right] + \Pi_{1c}D_4, \quad n = 1,2,3,...,a-1 \\ V_n &= \Pi_{21}\left[D_3 + e_3\frac{1-r^{a-1}}{1-r}\right] + \Pi_{1c}D_4, \quad n = a, a+1, ..., c-1 \end{aligned}$$

$$\begin{aligned} \text{where, } D_3 &= e_3\left\{D_1 + e_2r^a\left(\frac{1-(pe_1)^{c-a}}{1-pe_1}\right) + \left(\frac{e_2r^{a+1}}{pe_1 - r}\right)\left(\frac{pe_1 - (pe_1)^{c-a}}{1-pe_1} - \frac{r - r^{c-a}}{1-r}\right)\right. \\ &\quad \left. + \left(\frac{e_2r^{c+1}}{pe_1 - r}\right)\left(\frac{pe_1 - (pe_1)^{b-c}}{1-pe_1} - \frac{r - r^{b-c}}{1-r}\right)\right\} \\ D_4 &= e_3\left[\frac{1-(pe_1)^{b-c}}{1-pe_1} + D_2\right], \quad e_3 = \frac{\mu(1-\lambda)}{\lambda}. \end{aligned}$$

$$\begin{aligned} \Pi_{1c} &= \Pi_{21}D_5 \\ \text{where, } D_5 &= \frac{1}{1-e_1D_4}\left\{pe_1e_2r^{a-1}\left[(pe_1)^{c-1-a} + \left(\frac{r}{pe_1 - r}\right)\left((pe_1)^{c-1-a} - r^{c-1-a}\right)\right]\right. \\ &\quad \left. + e_1\left[D_3 + e_3\frac{1-r^{a-1}}{1-r}\right] + e_2r^{c-1}\right\} \end{aligned}$$

and

$$\begin{aligned} \Pi_{20} &= \Pi_{21}D_6 \\ V_0 &= \Pi_{21}D_7 \\ \text{where, } D_6 &= D_1 + D_2D_5 \\ D_7 &= D_3 + D_4D_5 \end{aligned}$$

Hence the steady state probabilities are,

$$V_0 = \Pi_{21} D_7 \tag{22}$$

$$V_n = \Pi_{21}\left[D_7 + e_3\frac{1-r^n}{1-r}\right], \quad n = 1,2,3,...,a-1 \tag{23}$$

$$V_n = \Pi_{21}\left[D_7 + e_3\frac{1-r^{a-1}}{1-r}\right], \quad n = a, a+1, ..., c-1 \tag{24}$$

$$\Pi_{1a} = \Pi_{21} e_2 r^{a-1} \tag{25}$$

$$\Pi_{1n} = \Pi_{21} e_2 r^{a-1}\left[(pe_1)^{n-a} + \frac{r}{pe_1 - r}((pe_1)^{n-a} - r^{n-a})\right], \quad n = a+1, a+2, ..., c-1 \tag{26}$$

$$\Pi_{1c} = \Pi_{21} D_5 \tag{27}$$

$$\Pi_{1n} = \Pi_{21}\left[D_5(pe_1)^{n-c} + e_2 r^c\left(\frac{(pe_1)^{n-c} - r^{n-c}}{pe_1 - r}\right)\right], \quad n = c+1, c+2, ..., b-1 \tag{28}$$

$$\Pi_{20} = \Pi_{21} D_6 \tag{29}$$

$$\Pi_{2n} = \Pi_{21} r^{n-1}, \quad n = 2,3,... \tag{30}$$

Then Π_{21} can be obtained by using the normalizing condition,

$$\sum_{n=0}^{c-1} V_n + \sum_{n=a}^{b-1} \Pi_{1n} + \sum_{n=0}^{\infty} \Pi_{2n} = 1$$

$$\begin{aligned}\therefore \Pi_{21} = {} & \left\{1 + D_5\left(1 + \frac{pe_1 - (pe_1)^{b-c}}{1 - pe_1}\right) + D_6 + cD_7 + r(1-r)^{-1}\right. \\ & + e_2\left\{r^{a-1}\left[1 + \left(\frac{pe_1 - (pe_1)^{c-a}}{1-pe_1}\right)\left(1 + \frac{r}{pe_1 - r}\right) - \left(\frac{r}{pe_1 - r}\right)\left(\frac{r - r^{c-a}}{1-r}\right)\right]\right. \\ & \left.\left. + \frac{r^c}{pe_1 - r}\left(\frac{pe_1 - (pe_1)^{b-c}}{1-pe_1} - \frac{r - r^{b-c}}{1-r}\right)\right\}\right. \\ & \left. + \frac{e_3}{1-r}\left[(a-1) + (1-r^{a-1})\left((c-a) - \frac{r}{1-r}\right)\right]\right\}^{-1}\end{aligned} \tag{31}$$

3.2. Expected Queue Length

In this model the expected queue length L_q is given by

$$\begin{aligned}L_q &= \sum_{n=1}^{c-1} n.V_n + \sum_{n=1}^{\infty} n.\Pi_{2n} \\ &= \Pi_{21}\left\{D_7\frac{c(c-1)}{2} + (1-r)^{-2} - cr^{a-1}\left(a + \frac{c-1}{2}\right)\right. \\ &\quad \left. + \frac{e_3}{1-r}\left[\frac{c(c-1)}{2} - r(1-r)^{-2} + (1-r)^{-1}\left(ar^{a-1} - r^{a+1}\right)\right]\right\}\end{aligned} \tag{32}$$

3.3. Determination of the Optimal Control Limits

The optimal control limits can be obtained by minimizing expected cost function. Here we assume that costs are charged for serving the customers and for holding them in the queue. Let C_0 be the over

head cost to initiate a batch service, C_h be the holding cost per customer in the queue in steady state, C_1 be the cost per batch in the case of accessible batch service and C_2 be the service cost per batch in the case of non-accessible batch, independent of batch size. Here it is convenient to take $C_1 = C_2$, since we adopt accessibility under customer's choice.
The expected cost function is given by

$$\begin{aligned}
E(cost) &= C_0 + C_h L_q + C_1 \sum_{n=a}^{b-1} \Pi_{1n} \\
&\quad + \sum_{k=0}^{\infty} \left\{ \sum_{n=0}^{b-1} (kC_2 + C_1) + \sum_{n=b}^{d} (k+1)C_2 \right\} \Pi_{2,kd+n} \\
&= C_0 + C_h L_q + C_1 \Pi_{21} \left\{ D_5 \left(1 + \frac{pe_1 - (pe_1)^{b-c}}{1 - pe_1} \right) \right. \\
&\quad + e_2 \left[r^{a-1} \left\{ 1 + \left(\frac{pe_1 - (pe_1)^{c-a}}{1 - pe_1} \right) \left(1 + \frac{r}{pe_1 - r} \right) - \left(\frac{r}{pe_1 - r} \right) \left(\frac{r - r^{c-a}}{1 - r} \right) \right\} \right. \\
&\quad \left. \left. + \frac{r^c}{pe_1 - r} \left(\frac{pe_1 - (pe_1)^{b-c}}{1 - pe_1} - \frac{r - r^{b-c}}{1 - r} \right) \right] \right\} \\
&\quad + \sum_{k=0}^{\infty} \left[\sum_{n=0}^{b-1} (kC_2 + C_1) + \sum_{n=b}^{d} (k+1)C_2 \right] \Pi_{2,kd+n} \qquad (33)
\end{aligned}$$

4. Conclusion

An (a, c, d) policy discrete time bulk service queue with accessible and non-accessible batches under customer's choice has been carried out in this article and expressions for steady state probabilities and expected queue length are obtained. It can be noted that the value of p affects the control limit.

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